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Application of Mathematical Optimization Techniques in Reservoir Design and Management Studies

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APPLICATION OF MATHEMATICAL OPTIMIZATION TECHNIQUES IN RESERVOIR DESIGN AND MANAGEMENT STUDIES

by

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ABSTRACT

Optimal monthly release policies are derived for Hoover Reservoir, Columbus, Ohio, using chance-constrained linear programming and dynamic programming-regression methodologies. Simulation procedures are used to examine and compare the overall performance of the optimal policies derived by the two methods. Results suggest that for a two-sided quadratic loss function, linear release policies are more optimal. It is also established that the maximum R^2 criterion, generally used for model selection, does not exactly produce the best form of a release policy, particularly for nonlinear forms. At target releases at or below the safe yield of the case study reservoir, and for a one-sided quadratic loss function, the standard policy is optimal. At higher targets, nonlinear policies give better performance than the standard policy. Other observations are made concerning the performance of the two optimization approaches in a real case study.

PROJECT PERSONNEL

Name	Project Position	Department
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LIST OF SYMBOLS

Symbol	Definition
a_m	Ratio of minimum storage, m_i to the reservoir capacity, C;
B_o, B_1, B_2	Regression coefficients;
b_i, b_{i-1}	Decision constants;
C	Reservoir capacity;
C_v	Coefficient of variation (m_X/σ_X);
CRP	Cross product of reservoir storage at beginning of any period, and inflow in the same period;
D	Maximum departure of sample probability distribution function, F_n (x), from the theoretical distribution, F(x);
D*	Kolmogorov - Smirnox statistic (critical value for D);
DP1, DP2, DP3	Dynamic programming - regression release policies;
dh	Potential drop (change in piezometric head);
dx	Distance over which the potential drop, dh, is observed;
E	Expected value operator;
FMAX	Maximum flood control reservoir storage;
F(X)	Hypothesized theoretical probability distribution;
$F_n(x)$	Sample probability distribution function of random variable, x, defined by a random sample of size n;
$F(R_i)$	Probability distribution function of random inflows, R_i, in month i;
$F^\alpha(R_i)$	Cumulative probability of random inflow, R_i, in month i equal to reliability α;

Symbol	Definition
f_i	Maximum release permissible in month i;
$f(x)$, $f(y)$	Probability density function of random variables x and y, respectively;
$f_i(s_i)$	Optimal return at stage i of the dynamic programming algorithm corresponding to beginning reservoir storage, s_i;
$f(x_i, x_2)$, $f(y_1, y_2)$	Bivariate joint probability density functions of random variables, x_1, x_2 and y_1, y_2, respectively;
$g(x)$	Probability density function of random variable, x;
$g(x_1, x_2)$	Bivariate joint probability density function of random variables x_1 and x_2;
i	Time period (month or year) or a particular stage in the dynamic programming algorithm;
j	Month;
K	Permeability;
K_j	Modular coefficient;
$L_i(S_i, X_i)$	Loss function;
LDR1, LDR2	Chance-constrained programming release policies;
M1	Dynamic programming - linear regression release policy;
M2, M3	Dynamic programming - Non-linear regression release policy;
MDT	Mean detention time (C/μ);
$M(t)$	Moment generating function, where t is a parameter;
$M(t_1, t_2)$	Moment generating function of bivariate probability distributions, where t_1 and t_2 are parameters;
m_i	Minimum reservoir storage allowable in month i;

Symbol	Definition
N	Decision period;
n	Sample size;
p_j	Non-exceedance probability
QFL	Current reservoir inflow in month i;
QFL1, QFL2...QFL4	Lagged reservoir inflows in months i-1, i-2, i-3, i-4, respectively;
q	Seepage per unit length;
q_i	Minimum guaranteed release or flow in month i;
$q_{i,j}$, $q_{i,j-1}$	Inflows in year i and months, j and j-1, respectively;
R^2	Coefficient of determination;
REL	Reservoir release in month i;
R_t, R_{t-1}	Reservoir inflows in periods t and t-1, respectively;
R_i, R_{i-1}	Reservoir inflows in months i and i-1, respectively;
$r_i(S_i, X_i)$	Economic return function;
r_i^α	α - percentile inflow in month i;
S_t, S_{t-1}	Reservoir storages at the end of periods t and t-1, respectively;
S_i, S_{i-1}	Reservoir storages at the end of months i and i-1, respectively;
STG	Reservoir storage at the beginning of month i;
SMAX	Maximum usable storage (for conservative purposes) in the reservoir;
SMIN	Minimum available storage (for conservative purposes) in the reservoir;

Symbol	Definition
S_N^*, S_0^*	Initial and final reservoir storage conditions, used in the dynamic programming algorithm, respectively;
SUM1	Sum of current inflow and beginning storage in month i (QFL + STG);
SUM2	$(QFL + STG)^2$
SUM3	$(QFL + STG)^3$
t_1, t_2, t_i, t_j	Standard normal deviates;
t	Time period;
T, T_t	Target levels;
$t_i(S_i, X_i)$	Transformation function at stage i of dynamic programming algorithm.
u_1, u_2	Uniform random variables;
Var	Variance operator;
V_i	Maximum flood control reservoir storage in month i;
w	Width of seepage face;
w_i	Weighting factor;
X_t, X_{t-1}	Reservoir release in periods t and t-1, respectively;
X_i, X_{i-1}	Reservoir release in months i and i-1, respectively;
X_i^*	Optimal reservoir release as obtained by the dynamic programming algorithm;
x, x_1, x_2	Random variables;
y, y_1, y_2	Random variables;
λ, λ_i	Parameters;

Symbol	Definition
$\rho_{x,y}$	Correlation coefficient of random variables x and y;
$\rho_{x1,\ x2},\ \rho_{y1,\ y2}$	Correlation coefficient of random variables x1, x2 and y1, y2, respectively;
ρ_j	Correlation coefficient of inflows in months j and j-1;
ρ	Population Lag-1 correlation coefficient of inflows;
$\hat{\rho}_x$	Estimated lag-1 correlation coefficient of inflows;
μ	Population mean of inflows;
$\hat{\mu}_x$	Estimated mean of inflows;
$\mu_{x1,\ x2},\ \mu_{y1,\ y2}$	Expected values of the joint probability distribution of random variables x_1, x_2 and y_1, y_2 respectively;
$\mu_j,\ \mu_{j-1}$	Population mean flows in months j and j-1, respectively;
σ	Population standard deviation of inflows;
$\hat{\sigma}_x$	Estimated standard deviation of inflows;
$\sigma_{x1,\ x2},\ \sigma_{y1,\ y2}$	Standard deviations of the joint probability distribution of random variables x_1, x_2 and y_1, y_2, respectively;
$\sigma_j,\ \sigma_{j-1}$	Population standard deviations of inflows in months j and j-1, respectively;
η_x	Coefficient of variation (σ_x/μ_x);
γ_x	Population skewness coefficient of inflows;
$\hat{\gamma}_x$	Estimated skewness coefficient of inflows;
$B_0,\ B_1,\ldots B_{12}$	Regression coefficients;
α	Reliability level imposed on all chance-constraints in month i;

Figure	Definition
α^*	Reliability level associated with chance-constraints imposed on the maximum storage, minimum storage,and maximum release levels in month i;
α'	Reliability of meeting the minimum guaranteed flow, q_i, in month i;
α_1, α_2, α_3, α_4	Reliability levels imposed on maximum storage, minimum storage, minimum release and maximum release levels, respectively*;
Γ	Gamma function;

* α^* is similar to α_1, α_2, and α_4 when the latter are set at the same level. α' and α_3 are identical.

Chapter I

INTRODUCTION

Mathematical optimization techniques have been widely used to analyze reservoir systems. Linear programming, dynamic programming, nonlinear programming and simulation procedures have all been used to formalize design and operation. Extensive efforts were directed toward determining optimal release policies for single, multi-purpose reservoir systems operated under various operational and physical constraints. The practitioner is not prepared to resort to these methodologies, however, for several reasons; lack of evidence as to the mathematical model's appropriateness in a real-world situation being a principal one. Mathematical models used to study a complex reservoir system should be supported by case studies to make them more meaningful and realistic. Comparison of various reservoir modeling techniques under a given real situation would facilitate the selection of the most suitable model as well as demonstrate the value of a mathematical programming approach. Therefore, the principal objective of this study is to investigate the application and comparison of mathematical optimization techniques such as linear, dynamic, and chance-constrained programming, as well as simulation techniques in the operation of a single, multi-purpose reservoir system. The study also attempts to derive general conclusions regarding the proper form of a monthly release policy for a representative, case example system. Some discussion of the nature of these monthly policies as related to the physical parameters of the reservoir system is also given.

Young (1966) used dynamic programming followed by regression analysis to derive annual operating policies for a single-purpose reservoir. Since the dynamic programming analysis was conducted using synthetically generated inflows, Young termed the method Monte Carlo dynamic programming. Young's conclusions are appropriate for annual reservoir operating policies under a quadratic, two-sided loss function. A similar approach is taken in this study to derive optimal operating policies for a single-purpose reservoir under both two-sided and one-sided quadratic loss functions, but for a monthly time scale. Since the monthly streamflow process is not stationary, the results obtained herein do not always correspond to those found by Young for the annual case.

It is also emphasized in this study that policies derived using the dynamic programming regression approach should be verified for their performance through simulation. A regression model with the highest coefficient of determination, R^2, may give the best fit to the data but produce less than optimal performance when used for actual operation. A trade-off between the coefficient of determination and the performance of a release policy is illustrated.

The form of the policy as derived from regression analysis and tested through simulation is important when multi-reservoir systems are examined, since dynamic programming cannot generally be used in such cases. Assumptions concerning the form of the policy allow other programming methods to be employed. One such procedure which has been used with some success is chance-constrained linear programming, as initially presented by ReVelle, et al. (1969). The procedure assumes that the optimal policy can be closely approximated by a release policy linear in the previous month's end of period storage or, equivalently, the previous month's inflow. To date, verification of this assumption has not been conducted for the case of a monthly time frame.

Optimal monthly release policies are derived using the chance-constrained linear programming approach. Such policies are verified and compared, through simulation, with policies obtained using the dynamic programming-regression methodology. Important characteristics of the chance-constrained linear programming approach are graphically illustrated and discussed.

Finally, conclusions are drawn as to the proper form of release policies for a single, multi-purpose reservoir, and the appropriateness of procedures tested in deriving these policies. Recommendations are made for further improvement in the approach to reservoir management.

Hoover Reservoir, on Big Walnut Creek in Central Ohio, is chosen as a case study site to provide realism to the analysis. The reservoir serves as a water supply source for the City of Columbus and also provides some flood control storage and recreational use. Reservoir storage volumes, in thousands of acre-feet, are about 60.3 for water supply, 25.8 for flood control surcharge; with a minimum storage of 2.2.

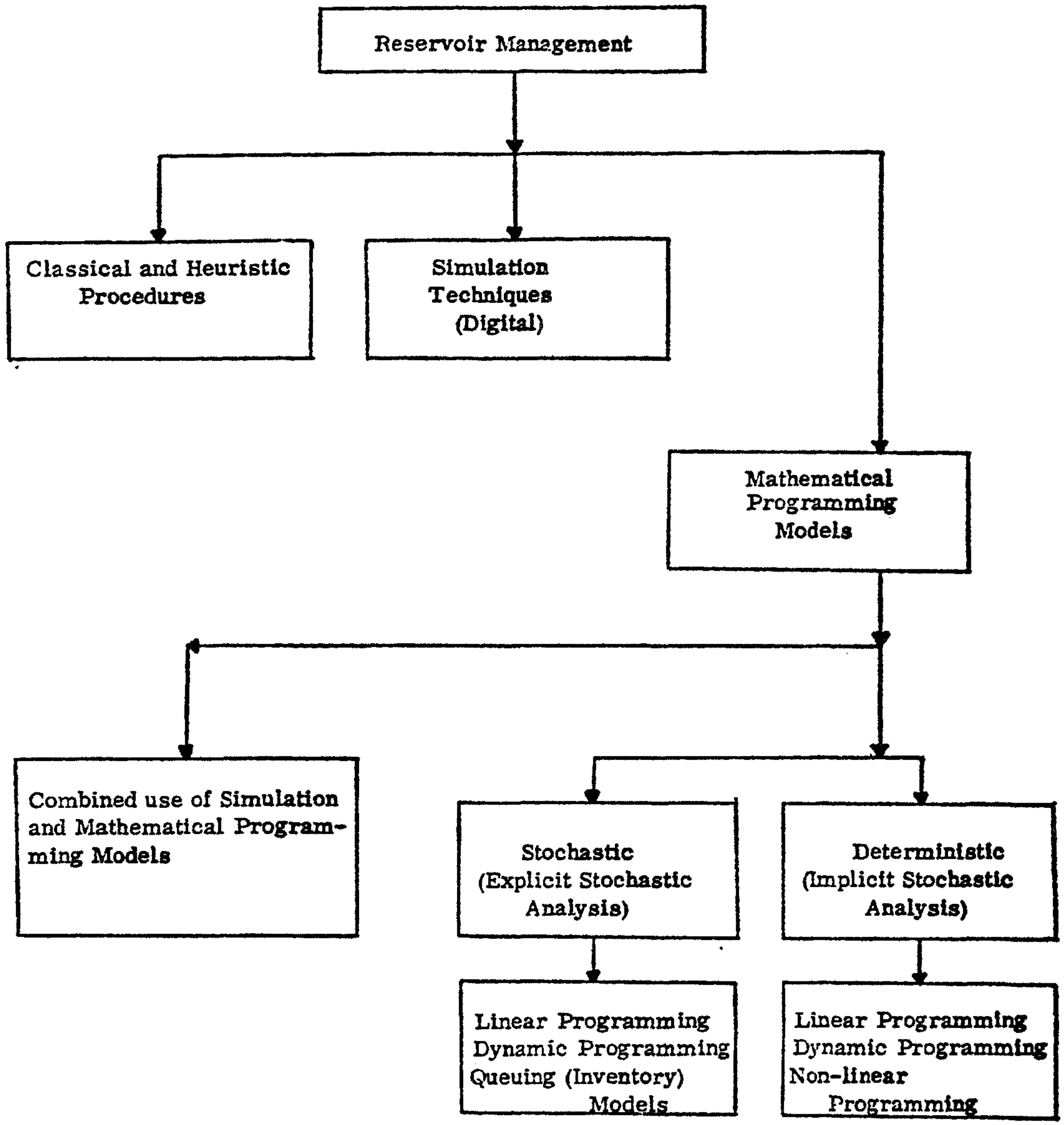
Chapter II

BACKGROUND AND OBJECTIVES

Classical Operating Policy: For a multi-purpose reservoir, the reservoir systems manager is faced with an intricate problem of how best to allocate storage and releases to meet the requirements of users. A classical operating policy as defined below, or policies similar to it, has been used where more refined methods are not available. The classical operating policy can be stated as (Roefs, 1968):

1. If there is not enough water to meet the target, release all water.
2. If there is more than enough water, release enough to meet target output, unless there is more water than can be stored, in which case the excess is also released.

The above policy, even for single-purpose reservoirs, may be far from the optimal use of storage available in the reservoir system. Mathematical models have evolved to devise an efficient and economical means of operating reservoirs. Figure 2-1 gives a representation of the general techniques employed to date in reservoir management studies. These are described in the following sections.



2.1 Schematic Diagram of Reservoir Optimization Procedures

Linear Programming Models and Single Reservoir Systems: A general description of this approach in the optimization of a single, multi-purpose reservoir is given in Roefs (1968). Under a deterministic environment, the reservoir inflows are specified and the mathematical programming problem amounts to determining optimal releases, over a specified period, subject to physical and operational constraints. One of the main problems encountered in this approach lies in the selection of an appropriate objective function to be optimized, as cost and benefit functions often are nonlinear. Piecewise linearization or separable programming may be used to reduce nonlinear objective functions to a convenient linear form (Hillier and Liberman, 1974). A good description of these procedures as applied to water resource systems is given in Windsor (1976), and Windsor and Chow (1972). Linear multiple regression is usually used with the deterministic approach to derive optimal operating rules.

Since streamflows are stochastic in nature, however, a linear programming approach incorporating probabilistic statements has gained considerable attention. One such approach is referred to as chance-constrained programming. Revelle, et al., (1969) illustrate this technique in their study of the optimal operation of a single multi-purpose reservoir. The idea behind chance-constrained programming is to specify the reliability with which operational constraints are to be imposed on the reservoir system. Thus reservoir system managers can make commitments on the maximum and minimum releases from the reservoir with some probability of success in meeting these. Input to the mathematical model requires the specification of the probability distribution of inflows, which may be obtained from historical flows. Since storage and release depend on a random inflow, these quantities are probabilistic and their probability distributions are required if they are to be explicitly dealt with in the chance constraints. Revelle, et al., (1969) use a linear decision rule which expresses the release in any period as a function of end of previous period storage and a decision parameter. The advantage of using such a rule lies in the mathematical simplification it introduces. The chance constraints involve only the random inflow and the decision parameters (decision variables). This enables a deterministic equivalent of the chance constraints to be written as using the probability distribution of the inflows only. Comments by Loucks (1970) and Eisel (1970) on the use of the linear decision rule are valuable. Considerable changes have been introduced in the linear decision rule (Revelle, 1973, Loucks and Dorman, 1975) to make it as realistic as possible. For example, Loucks (1970) included an additional term, the current period's inflow in the linear decision rule as proposed by Revelle, et al., (1969), and demonstrated a reduction in the necessary reservoir capacity when solving a problem similar to the one used by Revelle, et al. In later papers, Revelle, et al., (1970,1975) apply the linear decision rule using various objectives which are general enough to be applicable to other mathematical programming models:

a. maximize the expected value of the weighted sum of storage or release commitments over all the periods,

b. minimize the expected value of losses due to variation of the releases from targeted values,

c. maximize the storage or release commitment with stated reliability, or

d. minimize the risk of having insufficient storage or releases.

In another probabilistic approach, Loucks (1968) considers the application of stochastic linear programming for a single reservoir subject to a set of serially correlated random inflows. A first-order Markov dependence between the inflows was assumed. The objective was to obtain the operating policy that would maximize the probabilities of transitioning to reservoir storage volumes that best meet the stated objectives.

A form of the objective function adopted by Loucks can be expressed as:

$$\text{maximize } Z = \sum_{vidt} \beta_{vidt} \cdot X_{vidt}$$

where X_{vidt} = joint probability of having an initial storage v, inflow i and discharge d in period t, and

β_{vidt} = net benefits in period t of maintaining a storage volume between v and v + i - d and discharging a volume d.

Due to lack of data on cost and benefit functions, Loucks used a different form of the objective function which minimizes the expected value of the deviations of actual releases and storages from their target levels.

By solving the stochastic linear programming problem, an evaluation is made of the probability distributions of actual reservoir volumes and discharges that would result if the optimal policy were followed. The decision variables are the unknown joint probabilities, X_{vidt}. The requirement of having a final storage v in period t equal to initial storage $\overline{v}$ in period t + 1 is a constraint on the problem and is based on the continuity equation. Markov models have been extended by Gablinger and Loucks (1970) to indicate the interrelationship between linear and dynamic programming approaches.

Linear Programming Models: Multi-Reservoir Systems: Le Clerc and Marks (1973) present a case study of Riviere du Nord, a tributary of Ottawa River, to demonstrate the application of the linear decision rule in conjunction with chance-constrained programming. The problem involves analysis of the joint operation of a network of reservoirs. The approach taken was to impose requirements on the minimum flow at a downstream point while considering recreation (a conflicting use of reservoir storage) in the objective function.

Thus their objective was to minimize the drawdowns in the reservoirs used for recreation while attempting to meet low-flow requirements. To overcome the nonlinearity of the drawdown objective function, a piecewise linearization technique was adopted. An observation made by these authors requires mentioning. They state that chance-constrained programming, although accounting for the randomness of the natural inflows, does not define the magnitude by which the system fails. This deficiency may be a serious problem since a small failure may be relatively unimportant, whereas a large failure may have long-term effects. A simulation of the reservoir system under the derived operating policy should give reasonable insight into the magnitude of this problem.

An elaborate analysis by Nayak and Arora (1971) of a network consisting of four reservoirs demonstrates yet another application of chance-constrained programming to multi-reservoir systems. The formulation is similar to the single reservoir model proposed by Revelle, et al., (1969) except for variations in the continuity equations and probability distributions of inflows. Their objective was to minimize the total capacity of the reservoir system while meeting certain performance standards. A sensitivity analysis was conducted to study the effect on the total system capacity and its distribution among the reservoirs. This was accomplished by varying the minimum storage levels, minimum and maximum flows to be released, and the required flood storage (freeboard).

Curry (1973) uses a chance-constrained approach to modelling a complex reservoir system without using the conventional linear decision rule. Releases are explicitly considered in the mathematical model. His objective was to minimize total operating costs, including pumping costs of diverted water. The mathematical complexity of not adopting the linear decision rule is reflected in the required determination of convoluted probability distributions of the random inflows.

Recognition of the special structure of a linear programming model of a complex reservoir system led to the application of decomposition techniques (Parikh, 1966, Windsor and Chow, 1972). The approach is to optimize designated subsystems individually before obtaining an overall optimization of the entire system. Such a procedure may not guarantee a global optimum.

Combined Use of Simulation and Optimization: The combined use of simulation and mathematical optimization models has been carried out by Jacoby and Loucks (1972). Mathematical models are used for preliminary screening of the best set of possible designs from various alternatives. Their problem poses an important question as to whether analytical methods currently available for optimizing a complex river basin system can yield comparable results to enable the best design alternatives to be screened out. A stochastic linear programming approach (Loucks, 1968) was used and coupled to a large-scale simulation of the Delaware River Basin. Details of the simulation approach are given in Hufschmidt and Fiering (1966).

Dynamic Programming (DP) - Single Reservoirs: This approach has two main advantages over the conventional linear programming approach: 1) nonlinear cost and benefit functions can be incorporated with no difficulty, and 2) the optimal solution may be expressed in a functional form. Sensitivity analysis using dynamic programming is not generally as attractive as in a linear programming model, however, since computer time may be increased in resolving the dynamic programming model. Early work carried out by Hall and Howell (1963) used synthetically-generated inflows (Fiering, 1967) as input to a deterministic dynamic programming model. Through several applications of the model, each with a different synthesized sequence of flows, they were able to determine the optimum reservoir capacity. A more complex case of a single, multi-purpose reservoir system was later solved using dynamic programming by Hall, Butcher and Esogbue (1968). The optimal operating policy was obtained under a complex set of constraints. Inflows again were deterministic. Butcher (1971) applied stochastic dynamic programming to derive the optimal operating policy for a reservoir using the conditional probabtility distributions of the inflows (a Markov model).

Young (1966) presents results on the optimal form of annual release policies for a hypothetical reservoir design, as derived utilizing a dynamic programming-regression technique. Annual inflows to a reservoir were generated by a Markov model, and dynamic programming was then applied to these inflows to derive optimal annual releases to minimize losses under alternative objective function forms. Regression analysis conducted on optimal releases indicated that, for a two-sided quadratic loss function centered on the target release, linear release policies were as good as, if not better than, nonlinear policies.

Literature Critique: In the previous section some of the existing mathematical optimization techniques, as adapted for reservoir design and management studies, were presented and discussed. An extensive review of these studies indicates that:

a. Young's (1966) conclusions, in his application of the Monte Carlo dynamic programming-regression methodology to the operation of a single reservoir system, are applicable for annual release policies. The extension and verification of this approach to a monthly time frame needs investigation. This is particularly important since most reservoir operation decisions are made on a seasonal, monthly, weekly, or even daily basis. Results from Young's studies of annual operating policies cannot be used for these shorter time frames due to the nonstationarity of the inflow process within a year time period. The form of optimal monthly release policies, therefore, needs to be determined.

b. In the case of both dynamic programming and linear programming approaches to reservoir management, case study results are lacking. Performance of the linear decision rule in the context of chance-constrained linear programming for actual reservoir design and operation has been particularly lacking. The literature does not record the application of the technique to any single, multi-purpose reservoir. Evaluation of the usefulness of the technique in an actual design/operation study, therefore, needs to be conducted.

c. A comparison, in terms of overall performance, of reservoir release policies derived using existing mathematical programming techniques is lacking. The linear decision rules proposed by Revelle, et al., (1969) and Loucks (1970) are generally recognized to be useful in preliminary multi-reservoir operation studies. A general consensus among researchers in the area of reservoir management, however, is that these policies, due to their mathematical simplicity, are not as favorable as alternative forms of release policies. Comparison of the linear decision rules with other release policies derived by existing mathematical optimization techniques, such as the dynamic programming-regression approach, requires further investigation.

For instance, the linear decision rule adopted by Revelle, et al., (1969), assumes that the release in any period is directly proportional to the reservoir storage at the beginning of the period. The rule also implies that the coefficient associated with the storage variable is equal to unity. It remains to be verified whether such a policy is optimal as compared to release policies which incorporate other variables and which may be of a different form.

Research Objectives: With the above considerations in mind, the present study has the following objectives:

1. To derive, using the dynamic programming-regression technique, optimal monthly release policies for a case study reservoir under alternative objective function forms, and to verify these results through simulation;
2. To investigate the performance of the linear decision rule, chance-constrained programming approach to reservoir design and operation for the same case study reservoir, verifying all conclusions through simulation studies; and
3. To search for improved forms of release policies that will satisfy the chance constraints imposed in 2. above, but which may further improve the objective function values achieved through chance-constrained programming. This will be done by utilizing knowledge gained under objective 1. above concerning the optimal form of monthly release policies, and by testing candidate policies for their reliability performance through simulation. Based

on these results, recommendations are made concerning the most likely form of these improved policies and of procedures that may be used to determine the parameters of such policies.

Chapter III

CASE-STUDY DESCRIPTION

Hoover Reservoir, located in Central Ohio, is used as a case example throughout the present study. The first section of this chapter provides a general description of the reservoir site. The next two sections discuss seepage and evaporation losses, while the final section is devoted to statistical analysis of streamflows prior to and after reservoir construction. Empirical probability distributions of monthly inflows are graphically illustrated. Appendices A-D supplement the discussion given in this chapter.

Site Description

Hoover reservoir is located on the Big Walnut Creek, 12 miles (19 Km) northeast of Columbus, Ohio. A drainage area of 190 square miles (492 Km^2) contributes an annual flow of 188.895 cfs (5.35 m^3/sec) at the reservoir site. The reservoir has been in operation since March, 1955, with the principal purpose of meeting the water supply needs for the City of Columbus. Reservoir capacity is 60,342 acre-feet with the crest of the spillway at elevation 890 feet (above M.S.L.). The lowest intake valve is located at an elevation of 842 feet, corresponding to a minimum available storage of 2,188 acre-feet. An additional storage of 25,7808 acre-feet is provided above the spillway crest through the use of tainter gates for flood-control purposes. Table 3-1 gives values of reservoir storage and surface area at selected elevations (Burgess and Niple Limited, undated).

Reservoir Seepage

Geologic profiles shown in Figures 3-1(a) and 3-1(b) indicate that the formation underlying Big Walnut Creek consists of a glacial drift, composed essentially of clayey till, overlying shale. Since such formations exhibit low permeabilities, in the range of 10^{-4} - 10^{-1} gpd/ft^2, it is reasonable to assume that reservoir seepage is minimal. Approximate estimates of net seepage into the reservoir are given in Appendix A, and show that ground water activity around the reservoir is insignificant from a water supply standpoint. Computational details are given in Appendix A.

Reservoir Evaporation

Reservoir evaporation can be computed using the procedure suggested by the U.S. Weather Bureau (Kohler,1955). Since this method requires solar radiation data which is unavailable at Columbus, lake evaporation data from Coshocton, Ohio are used in the analysis. The data are appropriately adjusted to account for the difference in the average annual evaporation between Columbus and Coshocton. Expected monthly evaporation values from Hoover Reservoir are presented in Table 3-2. Details of the computations are given in Appendix B.

Table 3-1: Reservoir Storage and Surface Area at Various Pool Elevations

Pool Elevation above Mean Sea Level (feet)	Reservoir Storage (acre-feet)	Reservoir Surface Area (acres)
819	0	0
820	1	2
825	46	16
830	214	51
835	690	140
840	1622	233
845	3037	333
850	5102	493
855	7880	618
860	11325	760
865	15566	936
870	20807	1160
875	27556	1539
880	36373	1988
885	47312	2387
890*	60342	2825
895	75501	3239
900**	93204	3843

*crest of spillway
**top of flood control tainter gates

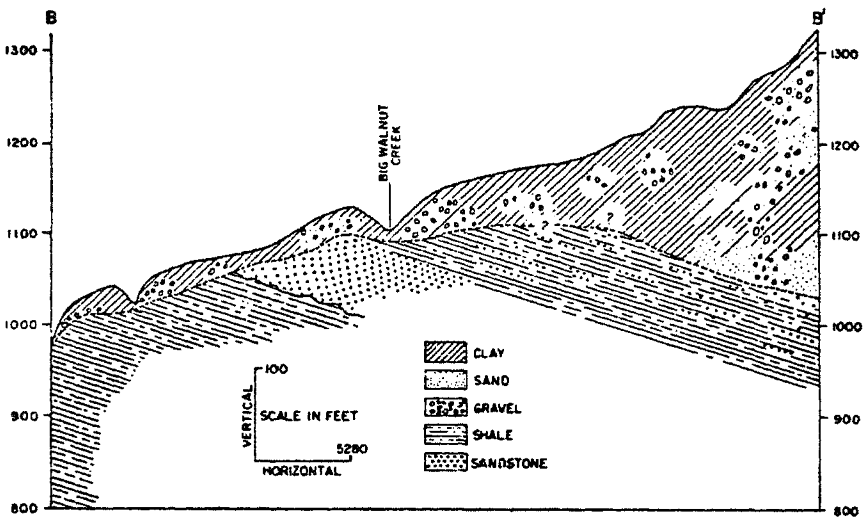


Figure 3.1 (a): Generalized Geologic Section in Northern Portion of Big Walnut Creek Basin

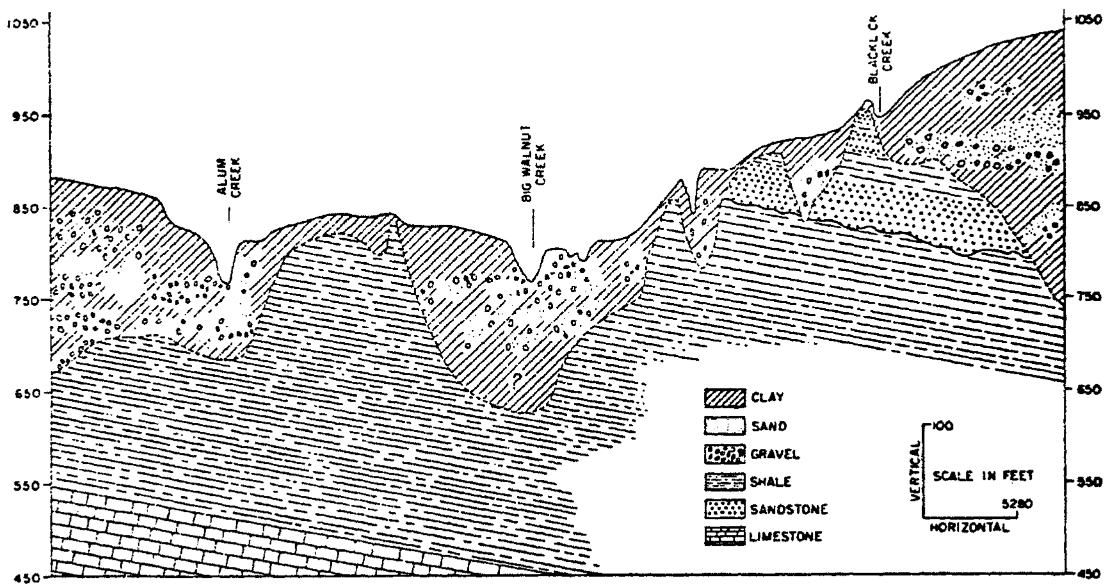


Figure 3.1 (b): Generalized Geologic Section Beneath the Southern Portion of Big Walnut Creek Basin

Table 3.2: Computed Average Monthly Evaporation From Hoover Reservoir

Month	Evaporation (in inches)
January	0.624
February	0.909
March	1.791
April	3.087
May	4.456
June	5.229
July	5.529
August	4.778
September	3.360
October	2.079
November	1.044
December	0.614

Streamflow Data Analysis

Streamflow Statistics: Natural streamflow records for the Big Walnut Creek are available over a relatively short period of 16 years (1939-1954) prior to the construction of the reservoir in September, 1954. In order to determine the effect of the reservoir on the streamflow regime, observed reservoir releases, measured at a gage located 0.5 miles downstream, are routed backwards to compute the inflows. Observed monthly releases and storage levels recorded over the period 1955-1973 by the U.S. Geological Survey, as well as monthly evaporation estimates, were used in the routing procedure. Important statistics based on the monthly natural and computed inflows are shown in Tables 3-3 and 3-4. Statistics on combined data of the natural and computed inflows are also presented for comparison. Statistical significance tests are applied to compare the mean and variance estimates of the natural and computed flows. Results from these tests (refer to Appendix

Table 3-3: Statistics of Natural Inflows*

Month	Mean in cfs ($\hat{\mu}_x$)	Standard Deviation in cfs ($\hat{\sigma}_x$)	Coefficient of Variation ($\eta_x = \hat{\sigma}_x/\hat{\mu}_x$)	Skewness Coefficient ($\hat{\gamma}_x$)	Lag-one Correlation Coefficient ($\hat{\rho}_x$)
January	361.426	386.729	1.070	1.053	0.498
February	378.112	210.353	0.556	-0.331	0.351
March	400.069	235.231	0.588	1.027	0.410
April	325.681	185.343	0.569	-0.022	0.380
May	160.931	136.260	0.847	1.463	0.032
June	201.823	206.680	1.024	1.278	0.717
July	94.685	97.337	1.028	1.082	0.644
August	42.166	112.629	2.671	3.515	0.479
September	16.991	44.757	2.634	3.422	0.152
October	11.119	10.301	0.926	0.753	0.166
November	71.539	68.217	0.954	0.404	0.518
December	193.459	209.353	1.082	0.932	0.815

*Based on a sample size, n = 16.
(1 cfs = 0.0283 cu m/sec)

Table 3-4: Comparison of Statistics

Month	Means (cfs)			Standard Deviations (cfs)			Skewness Coefficients		
	Natural Flows	Computed Flows	Combined Flows	Natural Flows	Computed Flows	Combined Flows	Natural Flows	Computed Flows	Combined Flows
January	361.426	262.673	309.145	386.729	238.422	315.862	1.053	1.737	1.456
February	378.112	282.609	327.552	210.353	176.771	196.345	-0.331	0.465	0.110
March	400.069	431.266	416.585	235.231	308.831	273.010	1.027	1.106	1.153
April	325.681	393.266	361.461	185.343	266.794	231.203	-0.022	0.390	0.449
May	160.931	286.841	227.589	136.260	198.679	181.227	1.463	0.788	1.129
June	201.823	166.860	183.313	206.680	178.818	190.270	1.278	1.763	1.515
July	94.685	105.081	100.189	97.337	123.911	110.652	1.082	1.187	1.210
August	42.166	49.260	45.921	112.629	53.533	85.178	3.515	0.893	3.660
September	16.991	26.396	21.970	44.757	35.480	39.771	3.422	2.731	3.084
October	11.119	17.000	14.232	10.301	29.747	22.648	0.753	2.870	3.627
November	71.539	99.486	86.335	68.217	152.533	119.578	0.404	2.898	3.315
December	193.459	176.125	184.282	209.353	179.583	191.345	0.932	0.660	0.843

(1 cfs = 0.0283 m^3/s)

C) suggest that the natural and computed inflows are hydrologically similar. It will be shown in the next section that their probability distributions are also in close agreement.

Although the natural and computed inflows are statistically similar, computed inflows were not included in the simulation analysis, described in the next Chapter, due to probable error in the evaporation estimates. However to obtain a more stable estimate of the coefficient of variation, σ_x/μ_x, combined data is used in the months of August and September.

Probability Distributions: Empirical probability distributions of monthly streamflows can, in general, be identified by one of the following probability density functions.

1. Normal

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x - \mu)^2/2\sigma^2} \qquad (3-1)$$

for $-\infty \leq x \leq \infty$

2. Log-Normal

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\log x - \mu)^2/2\sigma^2} \qquad (3-2)$$

for $0 \leq x \leq \infty$

3. Gamma

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1}e^{-x/\beta} \qquad (3-3)$$

for $0 \leq x \leq \infty$

where,

x = monthly streamflow

$f(x)$ = probability density function

μ, σ = population mean and standard deviation, respectively

α = shape parameter

β = scale parameter

log = natural log function

$\Gamma(\alpha)$ = gamma function of α.

The normal distribution is symmetrical about the mean, μ, while the log-normal and gamma distributions exhibit skewness. For the log-normal distribution, the transformed random variable $y = \log x$ is normally distributed with mean μ_y and variance σ_y^2. The relationships between the parameters of the normally distributed random variable y and the log-normal distributed random variable x are derived in Appendix D-1. These relationships will be particularly useful in the discussion presented in the next Chapter.

The theoretical probability distributions shown in Equations 3-1, 3-2 and 3-3 are completely defined provided the parameters in each distribution are known. Where such information is unavailable the parameters may be estimated from the streamflow data. In order that these estimated parameters accurately represent the population parameters the following properties associated with the estimators are desirable (Markovic, 1965; Hogg and Craig, 1970).

1) Consistency - The probability that the absolute value of the deviation of estimator $\hat{\theta}$ from the population parameter θ is less than a small quantity, ε, tends to unity as sample size

n tends to infinity, i.e.,

$$\text{Prob} (| \hat{\theta} - \theta | < \varepsilon) \rightarrow 1 \text{ as } n \rightarrow \infty$$

2) Unbiasedness - The expected value of the estimator is equal to the population parameter, i.e.,

$$E(\hat{\theta}) = \theta$$

3) Efficiency - The estimator should have the smallest variance among all classes of consistent estimators.

The maximum likelihood method possesses the properties discussed above. In this procedure, the parameter, θ, of the probability density function $f(x,\theta)$ can be obtained by differentiating the natural log of the likelihood function, $L(x,\theta)$, with respect to θ and setting it to zero. The likelihood function is $\prod_{i=1}^{n} f(x_i,\theta)$ where $x_i (i = 1, 2...n)$ is the random sample drawn from the population with the probability density function $f(x, \theta)$. The maximum likelihood estimates of the parameters of the normal, log-normal, and gamma distributions are derived in Appendix D-2.

Table 3-5 gives the maximum likelihood estimates of relevant parameters for the natural streamflows at Hoover Reservoir. These values are computed using the expressions developed in Appendix D-2.

Having estimated the unknown parameters, the next step is to obtain the probability density function which best fits the given streamflow data. Several statistical tests are available to verify whether a set of observations can be attributed to a completely specified distribution function (cumulative probability distribution), $F(x)$. One such test is the Kolmogorov-Smirnov (K-S) test, described as follows. Consider a random sample $(x_1, x_2 \ldots x_N)$ with a distribution function, $F_n(X)$ such

Table 3-5: Maximum Likelihood Estimates of Distribution Parameters*

	Normal		Log-Normal		Gamma	
Month	$\hat{\mu}_x$ (cfs)	$\hat{\sigma}_x$ (cfs)	$\hat{\mu}_y$ (cfs)	$\hat{\sigma}_y$ (cfs)	$\hat{\alpha}$ (cfs)	$\hat{\beta}$ (cfs)
January	361.426	386.729	5.165	1.430	0.688	525.328
February	378.112	210.353	5.684	0.849	1.805	209.480
March	400.069	235.231	5.820	0.639	2.580	155.066
April	325.681	185.343	5.540	0.840	1.841	176.904
May	160.931	135.260	4.709	0.973	1.254	128.334
June	201.823	206.680	4.673	1.328	0.774	260.753
July	94.685	97.337	3.806	1.576	0.672	140.900
August	42.166	112.629	2.220	1.732	0.360	117.128
September	16.991	44.757	1.243	1.647	0.347	48.965
October	11.119	10.301	1.788	1.339	0.790	14.075
November	71.539	68.217	3.366	1.738	0.567	126.171
December	193.459	209.353	4.320	1.727	0.545	354.971

*Sample size n=16

that Prob $(X \leq x_n) = \sum_{i=1}^{n} x_n/N$. The problem is to determine whether the distribution $F_n(X)$ is statistically similar to the hypothesized theoretical distribution function $F(X)$. To do this the Kolmogorov-Smirnov statistic, D, is evaluated, where

$$D = \max_{X_i} \left[F(X_i) - F_n(X_i) \right] \qquad (i = 1, 2 \ldots n)$$

If the statistic, D, is less than the critical value, D^*, at a given level of significance, it can be concluded that the random sample $(x_1, x_2 \ldots x_N)$ is drawn from a population with distribution function $F(X)$. Table 3-6 gives critical values of D^* for the normal and gamma distributions, when the population mean and variance are unknown. Where the population distribution function is log-normal, the above test can be applied to the transformed random variable $y = \log x$, since y is normally distributed.

Finally, the theoretical and sample probability distributions for normal, log-normal and gamma are plotted using the estimated parameters (Table 3-5) and sample observations on natural inflows. The probability distributions of the computed inflows for normal and log-normal cases are also included for illustration. All distributions and the detailed procedures used to obtain them are given in Appendix D-3. Observed values of the Kolmogorov-Smirnov statistic, D, for each month and for each probability distribution are presented in Table 3-7. The corresponding critical values at the 5% level of significance are also given. On examining these results it may be concluded that:

Table 3-6: Critical Values of D^* for the K-S Test

	Significance Level, α		
1. Normal			
Sample Size, N	0.10	0.05	0.01
10	0.239	0.258	0.294
15	0.201	0.220	0.257
20	0.174	0.190	0.231
25	0.165	0.180	0.203
30	0.144	0.161	0.187
2. Gamma (with shape parameter, α = 1)			
10	0.293	0.321	0.378
15	0.242	0.267	0.314
20	0.211	0.232	0.277
30	0.174	0.191	0.225
3. Gamma (with shape parameter, α = 2)			
10	0.283	0.311	0.359
15	0.234	0.258	0.298
20	0.203	0.223	0.261
30	0.171	0.188	0.218
4. Gamma (with shape parameter, α = 3)			
10	0.274	0.299	0.351
15	0.228	0.249	0.293
20	0.200	0.218	0.259
30	0.165	0.180	0.212

Table 3-7: Kolmogorov-Smirnov Statistics, D

Month	Normal		Log-Normal		Gamma	
	Observed Value	Critical Value	Observed Value	Critical Value	Observed Value	Critical Value
January	0.237*	0.214	0.114	0.214	0.114	0.243
February	0.147	0.214	0.257*	0.214	0.237*	0.225
March	0.107	0.214	0.059	0.214	0.042	0.225
April	0.105	0.214	0.207	0.214	0.168	0.225
May	0.106	0.214	0.129	0.214	0.089	0.225
June	0.118	0.214	0.109	0.214	0.101	0.243
July	0.197	0.214	0.087	0.214	0.090	0.243
August	0.382*	0.214	0.157	0.214	0.172	0.243
September	0.334*	0.214	0.152	0.214	0.134	0.243
October	0.143	0.214	0.112	0.214	0.123	0.243
November	0.191	0.214	0.191	0.214	0.171	0.243
December	0.155	0.214	0.132	0.214	0.152	0.243

*fails K-S test

a) The normal distribution fails the K-S test in three months, while the log-normal and gamma distributions fail in only one month each.

b) Except in the high flow months of February and April, the log-normal or gamma distributions give a better fit to the observed streamflow data than the normal distribution.

c) The log-normal and gamma distributions yield about equal goodness-of-fit to the data.

d) Conclusion(b) is consistent with trends in the observed monthly skewness coefficients presented in Table 3-3; e.g., in the months of February and April the skewness coefficient is close to zero, a property of the normal distribution, while streamflows in the rest of the months exhibit a positive skew.

Chapter IV

CASE-STUDY SIMULATION ANALYSIS

Historical streamflow records, generally available over a short period of time, are usually insufficient to study the design and operation of reservoir systems. Therefore, the first section of this chapter describes a technique for generating streamflow sequences for longer periods of time than the historical sequence. Such a technique ensures that important statistical characteristics of the historical flows are maintained. Statistical tests on the generated sequences are also summarized to verify this. The final section is devoted to the study of the existing operation of Hoover Reservoir; the application of the sequent peak algorithm for determining the safe yield from the reservoir; and comparison of this result with the safe yield estimate found under a simulation environment utilizing a standard operation policy.

Streamflow Generation: Simulation studies, in conjunction with Monte Carlo* methods, are useful in examining the design and operation of a reservoir system. Since the behavior of such a system is chiefly governed by the random inflows, long streamflow records are required to enable a simulation study to be meaningful. It is generally accepted

*Monte Carlo method of analysis refers to the use of random or pseudo-random numbers to represent the behavior of a random process.

that most historical streamflow records are too short to be used alone in a simulation study. Longer streamflow sequences that maintain important characteristics of the historical streamflows can be generated using a procedure developed by Fiering (1971), termed "streamflow synthesis". The basic approach is to generate flows using a model of the general form:

$$q_i = d_i + e_i \tag{4-1}$$

where,

q_i = inflow in period i

d_i = deterministic component

e_i = random component

The deterministic component, d_i, incorporates both a mean value term as well as persistence, a property that reflects dependence between successive streamflows. Since streamflows can not be predicted with certainty, a random component, e_i, with mean zero and finite variance σ_e^2, is included. The probability distribution of this random component, e_i, depends on the theoretical probability distribution of the inflows. The component d_i may be expressed in an autoregressive form

$$d_i = \beta_0 + \beta_1(q_{i-1}) + \beta_2(q_{i-2}) \;\cdots\cdots + \beta_n(q_{i-n}) \tag{4-2}$$

where, (β_0, β_1 ... β_n) are regression coefficients and (q_{i-1}, q_{i-2}... q_{i-n}) are inflows in n previous periods or lags. The degree to which the current inflow, q_i, depends on the previous flows is measured by the correlation coefficient. Under the assumption that the current flow depends only on the inflow in the previous period, Equation 4-2 reduces to:

$$q_i = \beta_0 + \beta_1(q_{i-1}) \tag{4-3}$$

The above equation is designated as the Markov Model and is based on the Markov property that the present state of any system depends only on the previous state. A typical theoretical (Markovian) correlogram is illustrated in Figure 4-1. Furthermore, if the inflows q_i and q_{i-1} are assumed to be from a bi-variate normal distribution with mean μ and variance, σ^2, then it can be shown that (Hogg and Craig, 1970):

$$E(q_i \mid q_{i-1}) = \mu + \rho\,(q_{i-1} - \mu) \tag{4-4}$$

where, $E(q_i \mid q_{i-1})$ is the expected value of inflow q_i given the inflow in the previous period, q_{i-1}, and ρ is the correlation coefficient between the inflows q_i and q_{i-1}.

Although Equation 4-4 is a special case, it supplies a convenient form for a model that may be used to generate inflows which preserve important statistics of the historical inflows. Since Equation 4-4 resembles the Markov model, (Equation 4-3), the following form is proposed with the random component, e_i (Fiering, 1971):

$$q_i = \mu + \rho\,(q_{i-1} - \mu) + e_i \tag{4-5}$$

If the variance of the random component, e_i, is assumed to be $\sigma^2(1 - \rho)$ then Equation 4-5 can be used to generate inflows with mean μ and variance σ^2. Values for the random variable, e_i, can be obtained using

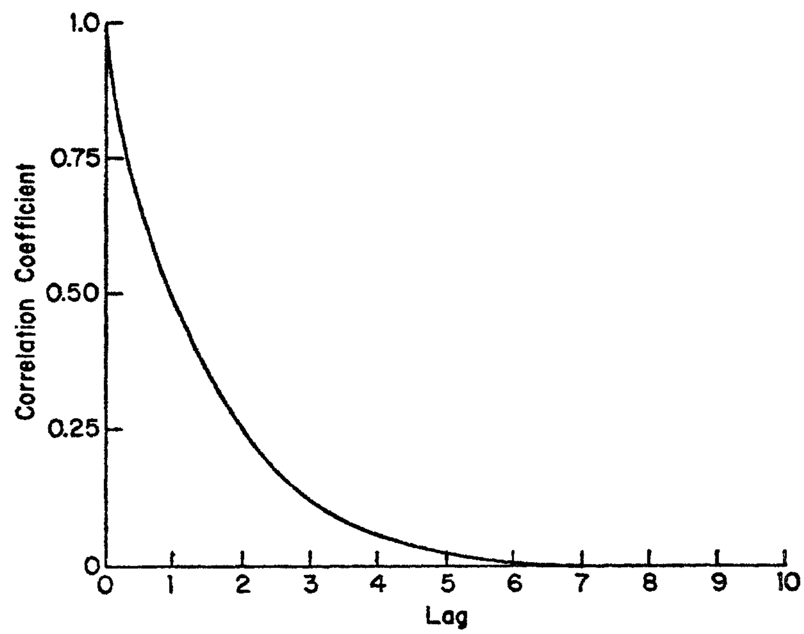


Figure 4-1 Theoretical Markovian correlogram

the linear transformation,

$$e_i = \sigma \cdot t_i\sqrt{1 - \rho} \tag{4-6}$$

where t_i is a standard normal deviate with mean zero and unit variance i.e. $t_i \sim N\ (0,\ 1)$.

Box and Muller (1958) suggest the following relationships between the standard normal deviates, t_i, and uniformly distributed random numbers. They are useful since computers generally provide pseudo-random numbers which are uniformly distributed.

$$t_1 = \sqrt{-2\log_e u_1}\quad \mathrm{Cos}(2\,\pi\, u_2)$$

and,

$$t_2 = \sqrt{-2\log_e u_1}\quad \mathrm{Sin}(2\,\pi\, u_2) \tag{4-7}$$

Using equations 4-5 and 4-6, the final form of the streamflow generating model can be expressed as

$$q_i = \mu + \rho\,(q_{i-1} - \mu) + \sigma \cdot t_i\sqrt{1 - \rho} \tag{4-8}$$

The above model is limited to generating annual flows. By a simple modification it can be extended to generate monthly streamflows. For the monthly case the generating model takes the form:

$$q_{i,j} = \mu_j + \rho_j \cdot \frac{\sigma_j}{\sigma_{j-1}}\,(q_{i,j-1} - \mu_{j-1}) + \sigma_j t_j\sqrt{1 - \rho_j} \tag{4-9}$$

where the indices i and j represent the year and month, respectively. The indices are related by $j = i(\text{mod } 12)$.

Table 4-1 illustrates the correlation of monthly flows with flows in previous months for the Big Walnut Creek, Hoover Reservoir site. These coefficients are computed using the historical inflows, prior to the construction of the reservoir. It may be concluded from the Table that in most of the months the current inflow is not significantly correlated with lagged inflows beyond the first lag. This conclusion is based on the assumptions that correlation coefficients associated with a significance probability greater than 0.05 can be ignored. This is in conformity with the Markovian property discussed earlier. Consequently, the use of Equation 4-9 to synthesize streamflow records for Big Walnut Creek, is justified.

In the previous chapter it was established that both log-normal and gamma probability distributions provide a good fit to the natural streamflows for Big Walnut Creek. Equation 4-9 can be readily adapted to generate flows that are derived from these two distributions. The generating procedure for the gamma distribution incorporates skewness explicitly in the random component, e_i (Fiering, 1971). Since in this study the sample of historical flows is small, estimates of skewness coefficients based on these flows are unstable. Thus, generating gamma flows using Equation 4-9 would be inaccurate and is, therefore, not considered in synthesizing streamflow records for the Big Walnut Creek.

To synthesize historical flows which are derived from a log-normal distribution, Equation 4-9 can be used to generate the

Table 4-1: Monthly Correlation Coefficients*

Month	Lags					
	1	2	3	4	5	6
January	0.498 (0.025)	0.458 (0.037)	0.058 (0.416)	-0.080 (0.393)	-0.222 (0.223)	-0.029 (0.461)
February	0.351 (0.092)	0.312 (0.120)	0.359 (0.086)	0.177 (0.256)	0.312 (0.138)	-0.410 (0.073)
March	0.410 (0.057)	-0.271 (0.155)	-0.023 (0.466)	-0.037 (0.446)	-0.092 (0.367)	-0.226 (0.200)
April	0.380 (0.073)	0.185 (0.247)	0.102 (0.353)	-0.167 (0.268)	-0.011 (0.484)	0.101 (0.355)
May	0.032 (0.453)	0.154 (0.284)	-0.115 (0.336)	0.012 (0.482)	0.067 (0.403)	0.194 (0.236)
June	0.717 (0.001)	0.004 (0.494)	0.040 (0.442)	-0.051 (0.426)	-0.050 (0.427)	-0.071 (0.396)
July	0.644 (0.004)	0.751 (0.001)	-0.147 (0.294)	0.098 (0.360)	-0.111 (0.341)	0.165 (0.271)
August	0.479 (0.030)	0.004 (0.494)	0.270 (0.156)	-0.336 (0.102)	0.182 (0.250)	0.007 (0.489)
September	0.152 (0.287)	0.711 (0.001)	0.694 (0.0001)	0.803 (0.0001)	0.105 (0.349)	-0.226 (0.200)
October	0.166 (0.277)	-0.090 (0.375)	0.072 (0.399)	0.445 (0.048)	0.158 (0.287)	-0.500 (0.029)
November	0.518 (0.020)	0.068 (0.405)	-0.243 (0.192)	-0.118 (0.338)	0.057 (0.420)	0.249 (0.185)
December	0.815 (0.0001)	0.221 (0.205)	-0.146 (0.302)	-0.274 (0.161)	-0.209 (0.228)	-0.112 (0.345)

*Numbers in the parantheses represent the significance probability of rejecting the null hypothesis, H_0, that the correlation is zero.

transformed variable, $y = \log_e x$. Although such a procedure preserves important statistics of the logs of the flows, it does not necessarily maintain the statistics of the original flows. According to Matalas (1967) this problem can be overcome by using the estimates of the mean, variance, skewness coefficient and the log-one correlation coefficient of the logs of the flows, obtained by solving the following equations:

$$\mu_x = e^{(\sigma_y^2/2 + \mu_y)} \tag{4-10}$$

$$\sigma_x^2 = e^{2(\sigma_y^2 + \mu_y)} - e^{(\sigma_y^2 + 2\mu_y)} \tag{4-11}$$

$$\gamma_x = \frac{(e^{(3\sigma_y^2)} - 3e^{(\sigma_y^2)} + 2)}{(e^{\sigma_y^2} - 1)^{3/2}} \tag{4-12}$$

$$\rho_x = \frac{(e^{(\sigma_y^2 \cdot \rho_y)} - 1)}{(e^{\sigma_y^2} - 1)} \tag{4-13}$$

where parameters indexed by x are statistics of the historical flows while those indexed by y are statistics of the normally distributed, log-transformed flows. The above relationships are derived for the two-parameter log-normal distribution in Appendix D-1 and used in the present study.

Statistics of the historical flows and the synthesized flows for the Big Walnut Creek are compared in Tables 4-2 and 4-3. Statistical significance tests indicate that the synthesized flows do preserve

Table 4-2: Comparison of Mean and Standard Deviation Estimates

Month	Historical Flows		Synthesized Flows*	
	Mean (cfs)	Standard Deviation (cfs)	Mean (cfs)	Standard Deviation (cfs)
January	361.426	386.729	345.811	374.162
February	378.112	210.353	378.792	211.844
March	400.069	235.231	395.487	226.474
April	325.681	185.343	332.702	146.574
May	160.931	136.260	181.424	154.330
June	201.823	206.680	195.595	188.360
July	94.685	97.337	92.325	100.525
August**	45.921	85.178	46.482	81.536
September**	21.970	39.771	20.524	29.467
October	11.119	10.301	10.914	9.905
November	71.539	68.217	70.395	65.701
December	193.459	209.353	190.396	219.872

*based on 1000 years of generated flows
**combined streamflow data prior to and after the construction of the reservoir are used to obtain more stable estimates of the variance.

Table 4-3: Comparison of Lag-one Correlation and Skewness Estimates

Month	Historical Flows			Synthesized Flows**	
	Lag-One Correlation Coefficient	Skewness Coefficients Observed	Skewness Coefficients Theoretical*	Lag-One Correlation Coefficient	Skewness Coefficient
January	0.498	1.053	4.435	--	4.064
February	0.351	-0.331	1.841	0.391	1.746
March	0.410	1.027	1.967	0.437	1.600
April	0.380	-0.022	1.892	0.393	2.051
May	0.032	1.463	3.147	0.071	2.991
June	0.717	1.278	4.146	0.731	2.991
July	0.644	1.082	4.170	0.632	4.604
August	0.479	3.660	11.948	0.511	5.557
September	0.152	3.084	11.360	0.098	3.906
October	0.166	0.753	3.575	0.229	2.964
November	0.518	0.404	3.728	0.511	2.974
December	0.815	0.932	4.514	0.753	4.375

*The theoretical skewness coefficient can be estimated using the relationship $(\eta^3 + 3\eta)$ where η is the coefficient of variation and is given by the ratio $\hat{\sigma}_X/\hat{\mu}_X$.

**Estimates of these statistics are based on 1000 years of generated data.

important statistics, such as the mean and variance, of the historical flows. Although the skewness coefficients of the generated flows show marked differences from those of the historical flows (observed skewness coefficients), this should not be critical since the estimates of these coefficients, based on the small sample of historical flows, are unstable. The theoretical skewness coefficient is also well preserved by the generated flows. It must be emphasized again that the use of Equation 4-9 to generate log-normal flows does not require an estimate of the skewness coefficient. Fiering (1971), in his study of streamflow synthesis, states:

> "Empirical studies have shown that the mean and standard deviation are much more important than other statistics in producing good results in most basin simulation studies. Other statistics, such as higher order moments (skewness coefficient, for example) and correlation coefficients for larger lags are subject to more pronounced sampling errors; therefore analysts may hesitate to use sample estimates. Fortunately these less stable statistics do not seem crucial for rational evaluation of alternative schemes" (p. 36).

Study of the Existing Policy: Since the operation of Hoover reservoir began in September, 1955, the draft rate has increased from an annual average of 19.72 M.G.D. in 1957 to 75.0 M.G.D. in 1977. The varying draft suggests that the operation was mainly directed toward meeting Columbus water supply demand each year. Inquiry as to specific operating policies has shown that no pre-determined operating rules are utilized. Lacking a definite policy statement, it is assumed that the operating policy corresponds to the standard policy, shown in Figure 7-4. This assumption is tested through a simulation experiment based upon historical data and the assumed standard operating policy.

Computed inflows derived from observed releases, storages and computed average monthly evaporation, over the period 1957 to 1972, are used in the analysis. Statistical results from the simulations are presented in Tables 4-4 and 4-5. Statistical tests on the mean and variance of the monthly storages and releases under the two policies, indicate that they are similar at a 5% level of significance. Also, the statistics on average reservoir surface area and average evaporation as presented in Table 4-5 are in very close agreement under the two policies. Thus, the observed similarity between the two policies suggests that the past operation of Hoover reservoir has followed a standard policy. This policy can then be used to examine the future performance of Hoover Reservoir in meeting the current (1977) draft of 75.0 M.G.D.

Future Operation of Hoover Reservoir: The streamflows generated according to the procedure discussed earlier in this chapter are used to simulate the operation of Hoover Reservoir under a standard policy. Performance characteristics based on 20 simulation runs, each over a period of 148 years, are presented in Table 4-6. All measures are computed monthly averages for the total period of simulation, and incorporate the current design specification of Hoover Reservoir (See Chapter 3).

Average evaporation values, shown in Table 3-2 of Chapter 3, are used in the simulation to account for this loss in yield. Evaporation from the reservoir in any month is computed by multiplying the average evaporation, (expressed in feet) by the average reservoir surface area. The average reservoir surface area is derived using the average storage observed in that month in conjunction with linear interpolation between

Table 4-4: Average Monthly Storage and Release for the Period 1957-1972 (All units expressed thousand acre-feet)

Month	Existing Policy		Standard Policy	
	Average Storage	Average Release	Average Storage	Average Release
January	45.026	10.019	47.147	12.071
February	51.590	11.583	51.657	10.794
March	55.016	20.213	55.871	21.799
April	60.581	22.643	59.847	23.028
May	60.548	16.608	59.432	16.485
June	59.865	8.576	58.875	8.363
July	57.372	7.709	56.595	6.556
August	53.524	5.412	53.889	4.903
September	49.633	4.804	50.490	4.229
October	45.516	4.704	47.003	4.284
November	41.753	6.278	43.723	5.932
December	41.566	7.891	43.881	8.136

Table 4-5: Average Reservoir Surface Area and Evaporation

Month	Existing Policy		Standard Policy	
	Average Reservoir Surface Area (th. acres)	Average Evaporation (th. ac. ft.)	Average Reservoir Surface Area (th. acres)	Average Evaporation (th. ac. ft.)
January	2.394	0.125	2.434	0.127
February	2.564	0.191	2.587	0.193
March	2.736	0.397	2.747	0.400
April	2.828	0.705	2.813	0.703
May	2.816	1.045	2.803	1.042
June	2.763	1.185	2.757	1.185
July	2.659	1.186	2.674	1.196
August	2.527	1.002	2.567	1.020
September	2.387	0.637	2.440	0.655
October	2.242	0.383	2.313	0.396
November	2.160	0.177	2.242	0.185
December	2.235	0.115	2.313	0.119

Table 4-6: Simulation Results under a Standard Policy: 75.0 M.G.D. Draft

Month	Average Release (th. ac. ft.)	Average Inflow (th. ac. ft.)	Average Storage (th. ac. ft.)	Average Evaporation (th. ac. ft.)	Percentage Shortages per month
January	14.196	21.904	31.260	0.096	1.72
February	13.020	20.805	38.872	0.163	0.14
March	18.250	24.403	46.494	0.361	0.00
April	16.523	19.588	52.286	0.665	0.00
May	11.010	11.288	54.686	0.974	0.00
June	11.833	11.961	53.991	1.132	0.07
July	8.304	5.805	52.987	1.159	0.17
August	7.600	2.877	49.328	0.932	0.51
September	6.860	1.246	43.673	0.591	1.55
October	6.976	0.679	37.469	0.325	3.24
November	6.772	4.254	30.847	0.141	4.12
December	8.750	11.884	28.188	0.083	3.21

the appropriate storage levels as specified by the storage-surface area relationship (See Table 3-1, Chapter 3).

From Table 4-6, the maximum percentage of shortages occurs in the month of November. A 4.12% shortage is equivalent to 6 shortages in 148 years or approximately 2 shortages in 50 years. Thus the maximum probability of failure to meet a draft of 75.0 M.G.D. in any month is 0.04.

Table 4-7 illustrates the average yearly shortages for various draft levels. These results show that, under a draft of 75.0 M.G.D., the average number of yearly shortages over the period of 148 years is

Table 4-7: Statistics of Yearly Shortages

Draft (M.G.D.)	Average Number of Yearly Shortages in 148 Years	Probability of Shortage in a Year
68	3.00	0.020
69	3.75	0.025
70	4.30	0.029
73	7.05	0.048
75	9.10	0.062

9.10. This implies that the probability of a shortage in any year is 0.062. The Table also shows that the safe yield from the reservoir is 68 M.G.D.* and is associated with a 0.02 probability of having a shortage in a year or equivalently 1 shortage in 50 years.

A comparison of this estimate of safe yield with that derived using the sequent peak algorithm is made in the next section.

*This estimate excludes losses due to evaporation.

Estimated Safe Yield Using Sequent Peak Algorithm: Given an inflow and draft pattern, the sequent peak algorithm yields the minimum storage required to just meet the required draft over the period of analysis. The method can be shown to be equivalent to the Rippl method of computing required storage, but is more easily programmed for computer computations. In this study the sequent peak algorithm is solved for 60 generated flow sequences of 50 years length each. Statistics on required storage are derived based upon this sample size. For a 50 year flow sequence the sequent peak analysis gives the storage (for a given draft rate) that is associated with a probability of exceedence* equal to 0.02 or equivalently this storage will be exceeded once in 50 years. In other words under a given draft the probability of shortage in any year is 0.02. Since the 50-year inflow sequences are random, the maximum storage is also random. Table 4-8 illustrates the storage derived by the sequent peak algorithm at selected draft levels. The storage-yield curve based on these results is shown in Figure 4-2.

The draft rates assumed in the sequent peak analysis include losses from the reservoir due to evaporation. Consequently, to obtain the available draft for a given storage, the evaporation loss must be subtracted from the drafts in Figure 4-2. The average evaporation (from Table 3-2, Chapter 3) is 5.90 M.G.D. From Figure 4-2 it is

*The probability of non-exceedence is computed by ranking the critical storages in the order of decreasing magnitude and applying the formula m/n+1 where m is the rank and n is a sample size of 50. Interpolation between storages associated with ranks 1/n+1 and 2/n+1 is used to obtain the storage at the 0.02 non-exceedence probability.

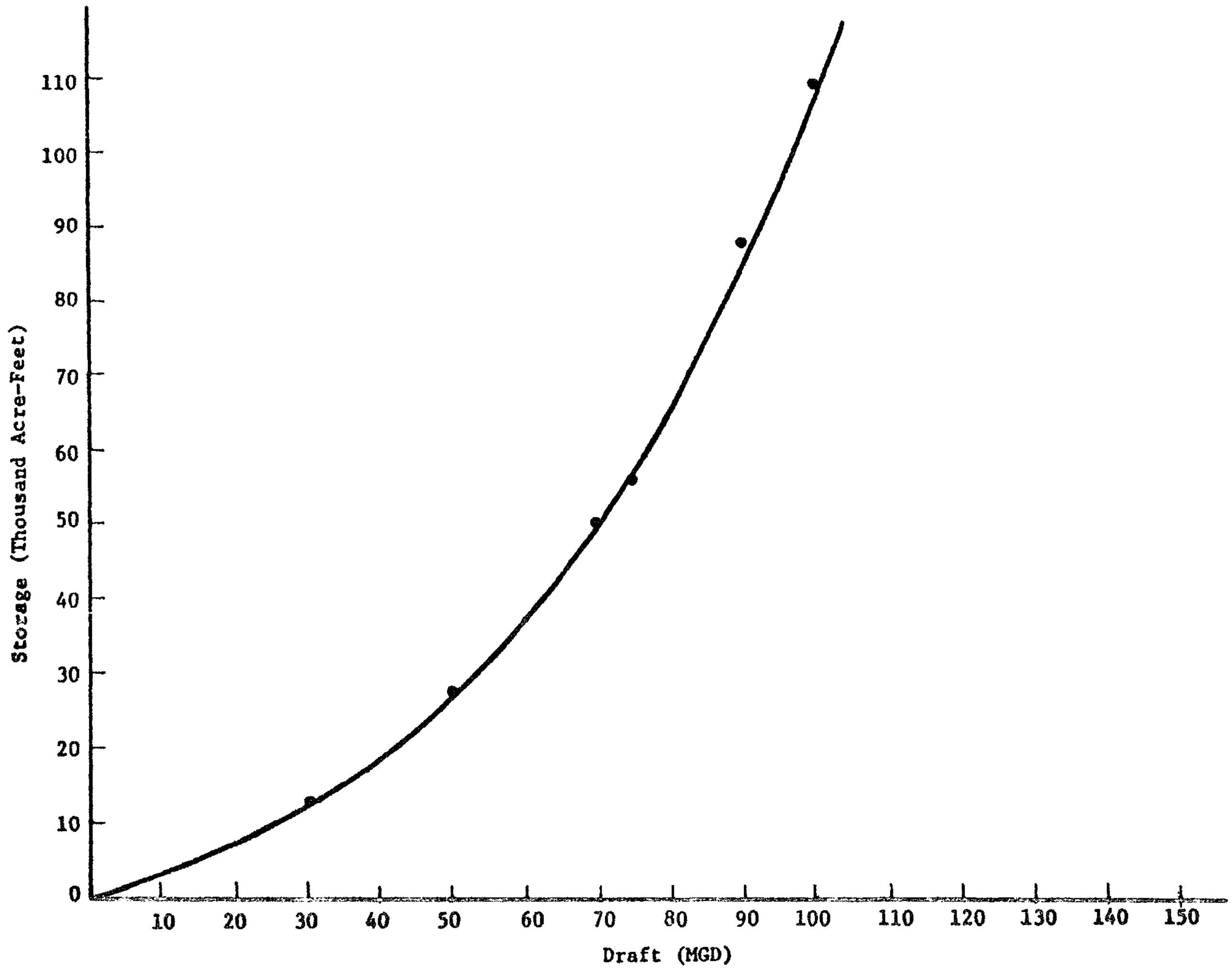


Figure 4-2 Storage-Yield Curve for Hoover Reservoir
(Non-exceedance Probability of 0.98)

Table 4-8: Required Storage for Various Draft Rates

Draft (M.G.D.)	Required Storage* (thousand acre-feet)			
	Upper 95% Confidence Limit	Mean	Lower 95% Confidence Limit	Standard Deviation
50	29.049	27.744	26.438	4.830
60	39.294	37.333	35.372	7.257
70	53.410	50.107	46.803	12.227
75	59.291	55.550	51.809	13.844
90	94.292	87.676	81.059	24.488

*Based on a 0.98 non-exceedance probability observed that the safe yield draft corresponding to the existing water supply storage at Hoover Reservoir, of 58,154 acre-feet is about 76.0 M.G.D. Thus, the available draft or the safe-yield from the reservoir, after excluding an average evaporation of 5.90 M.G.D., is approximately 70.0 M.G.D. According to the sequent peak analysis, the probability of shortage in meeting this draft is 0.02 or, equivalently, 1 shortage in 50 years.

In contrast, the simulation results under the standard policy, presented in Table 4-7, suggest that the probability of shortage in meeting the same draft is 0.029, and a safe yield from Hoover Reservoir is 68.0 M.G.D. The difference in these two figures is not great, and is probably due to the differential treatment of evaporation estimates under the two methods.

Chapter V

OPTIMIZATION MODELS

Mathematical techniques, available in operations research literature, can be applied to optimize the design and operation of a multipurpose reservoir system. Two such techniques which are currently proposed are presented in this Chapter. The first section describes the chance-constrained linear programming approach where constraints on the operation and design of the reservoir are probabilistic. Input requirements for this method are discussed and illustrated for the Hoover Reservoir case study. The final section is devoted to the dynamic programming-regression method. A recursive relationship is developed to obtain optimal releases from a reservoir.

Chance-Constrained Programming: This approach is similar to conventional linear programming methodology, except that the constraints in the problem are no longer deterministic. Instead, these constraints are expressed in a probabilistic form which is particularly useful when the state variables in the problem are stochastic.

Consider the problem of determining the minimum capacity of a single reservoir operated under certain physical and operational constraints. The constraints commonly imposed relate to restrictions on the reservoir release and storage levels. Since releases and storages are stochastic, it would be realistic to assign some degree of reliability to satisfying these constraints. This can be achieved by writing the following model (ReVelle, 1969):

$$\text{Minimize } C \qquad \text{5-1(a)}$$

subject to,

$$\text{Prob}\left\{S_t \leq C - v_i\right\} \geq \alpha_1 \quad (t = 1, 2 \ldots N) \qquad \text{5-1(b)}$$

$$\text{Prob}\left\{S_t \geq m_i\right\} \geq \alpha_2 \quad (t = 1, 2 \ldots N) \qquad \text{5-1(c)}$$

$$\text{Prob}\left\{X_t \geq q_i\right\} \geq \alpha_3 \quad (t = 1, 2 \ldots N) \qquad \text{5-1(d)}$$

$$\text{Prob}\left\{X_t \leq f_i\right\} \geq \alpha_4 \quad (t = 1, 2 \ldots N) \qquad \text{5-1(e)}$$

$$X_t, S_t \geq 0$$

where,

S_t = storage in period t;

X_t = release in period t;

v_i = flood-control storage in month i;

m_i = minimum required storage in month i;

q_i = minimum required release in month i;

f_i = maximum release permissible in month i;

α_1, α_2, α_3, and α_4 = reliability levels, or the probabilities of satisfying the constraints; and

N = length of the decision period.

In the above formulation, parameters indexed by i are cyclical over a year and are associated with a particular month, while variables indexed by t change over the time period under consideration. The two indices are related by the relationship

$$i = t(\text{mod } 12) \qquad (5\text{-}2)$$

The first two constraint sets, equations 5-1(b) and 5-1(c) ensure that adequate reservoir storage is available in each month for flood control and recreational purposes, respectively. The third constraint set (Equation 5-1(d)), guarantees a minimum release, q_i, in any month i, and the last set of constraints (Equation 5-1(e)) restricts the maximum release from the reservoir in order to prevent flood damages downstream.

Constraints of the above form are referred to as chance-constraints. In order to solve the chance-constrained linear program (Equations 5-1(a) - 5-1(e)), the chance-constraints have to be expressed in a deterministic form. This may be achieved by using the probability distribution function of the random variables X_t and S_t. Since these distributions are not defined unless the reservoir is operated using a particular release policy, the constraints on the release and storage must alternatively be expressed in terms of the inflow, R_t, in period t. To conveniently perform such a transformation, it is essential to adopt release policies similar to the linear decision rules proposed by ReVelle (1969) and Loucks (1970). The general form of a linear decision rule policy is:

$$X_t = S_{t-1} + \lambda_i R_t - b_i \tag{5-3}$$

where,

S_{t-1} = storage at the end of period t-1

R_t = inflow in period t

b_i = decision constant for month i

λ_i = parameter for month i $(0 \leq \lambda_i \leq 1)$

The continuity equation for reservoir storage at the end of any period t is given by

$$S_t = S_{t-1} + R_t - X_t \tag{5-4}$$

Substitution of Equation 5-3 in Equation 5-4 yields:

$$S_t = (1 - \lambda_i)R_t + b_i \tag{5-5}$$

or

$$S_{t-1} = (1 - \lambda_{i-1})R_{t-1} + b_{i-1} \tag{5-6}$$

Substituting Equation 5-6 in Equation 5-3 gives an alternate form of the release policy which is a function of the inflows only. Thus,

$$X_t = \lambda_i R_t + (1 - \lambda_{i-1})R_{t-1} + b_{i-1} - b_i \tag{5-7}$$

Using equations 5-5 and 5-7, the chance-constrained formulation becomes:

$$\text{Minimize } C \qquad \text{5-8 (a)}$$

subject to,

$$\text{Prob}\left\{(1 - \lambda_i)R_t + b_i \leq C - v_i\right\} \geq \alpha_1 \quad (t = 1, 2 \ldots N) \qquad \text{5-8 (b)}$$

$$\text{Prob}\left\{(1 - \lambda_i)R_t + b_i \geq m_i\right\} \geq \alpha_2 \quad (t = 1, 2 \ldots N) \qquad \text{5-8 (c)}$$

$$\text{Prob}\left\{\lambda_i R_t + (1 - \lambda_{i-1})R_{t-1} + b_{i-1} - b_i \geq q_i\right\} \geq \alpha_3 \qquad \text{5-8 (d)}$$

$$(t = 1, 2 \ldots N)$$

$$\text{Prob}\left\{\lambda_i R_t + (1 - \lambda_{i-1})R_{t-1} + b_{i-1} - b_i \leq f_i\right\} \geq \alpha_4 \qquad \text{5-8 (e)}$$

$$(t = 1, 2 \ldots N)$$

$$b_i \text{ unrestricted } (i = 1, 2 \ldots 12) \qquad \text{5-8 (f)}$$

If the probability distribution of the inflow, R_t, in any month i, is assumed to remain unchanged over the period N (strictly stationary process), the number of constraints in each equation of the above formulation will be reduced to 12 by replacing the index t by index i. Thus:

$$\text{Minimize } C \qquad \text{5-9 (a)}$$

subject to,

$$\text{Prob}\left\{ (1 - \lambda_i)R_i \leq C - v_i - b_i \right\} \geq \alpha_1 \quad (i = 1, 2...12) \qquad \text{5-9 (b)}$$

$$\text{Prob}\left\{ (1 - \lambda_i)R_i \geq m_i - b_i \right\} \geq \alpha_2 \quad (i = 1, 2...12) \qquad \text{5-9 (c)}$$

$$\text{Prob}\left\{ \lambda_i R_i + (1 - \lambda_{i-1})R_{i-1} \geq q_i - b_{i-1} + b_i \right\} \geq \alpha_3 \qquad \text{5-9 (d)}$$
$$(i = 1, 2...12)$$

$$\text{Prob}\left\{ \lambda_i R_i + (1 - \lambda_{i-1})R_{i-1} \leq f_i - b_{i-1} + b_i \right\} \geq \alpha_4 \qquad \text{5-9 (e)}$$
$$(i = 1, 2...12)$$

$$b_i \text{ unrestricted } (i = 1, 2...12) \qquad \text{5-9 (f)}$$

To write the deterministic equivalents of the chance-constraints, the probability distribution function of the linear tranforms $(1 - \lambda_i)R_i$ and $\lambda_i R_i + (1 - \lambda_{i-1})R_{i-1}$ must be obtained from inflow probability distributions $F(R_i)$ and $F(R_{i-1})$, respectively. However, since in this study the chance-constrained program is solved by setting λ_i to its extreme values of 0 and 1, the distribution function of the monthly inflows, $F(R_i)$ will suffice.

<u>ReVelle Formulation</u>: Setting λ_i equal to 0 in every month i, Equations 5-9 reduce to the chance-constrained formulation proposed by ReVelle (1969):

$$\text{Minimize } C \qquad \text{5-10 (a)}$$

subject to,

$$\text{Prob}\left\{ R_i \leq C - v_i - b_i \right\} \geq \alpha_1 \qquad (i = 1, 2\ldots12) \qquad \text{5-10 (b)}$$

$$\text{Prob}\left\{ R_i \geq m_i - b_i \right\} \geq \alpha_2 \qquad (i = 1, 2\ldots12) \qquad \text{5-10 (c)}$$

$$\text{Prob}\left\{ R_{i-1} \geq q_i - b_{i-1} + b_i \right\} \geq \alpha_3 \qquad (i = 1, 2\ldots12) \qquad \text{5-10 (d)}$$

$$\text{Prob}\left\{ R_{i-1} \leq f_i - b_{i-1} + b_i \right\} \geq \alpha_4 \qquad (i = 1, 2\ldots12) \qquad \text{5-10 (e)}$$

$$b_i \text{ unrestricted } (i = 1, 2\ldots12) \qquad \text{5-10 (f)}$$

The deterministic equivalents of the chance-constraints Equations 5-10 can be found by utilizing the distribution function, $F(R_i)$, shown in Figure 5-1. For a given reliability level, α, the quantities r_i^{α} and $r_i^{1-\alpha}$ represent the values of the random variable, R_i, such

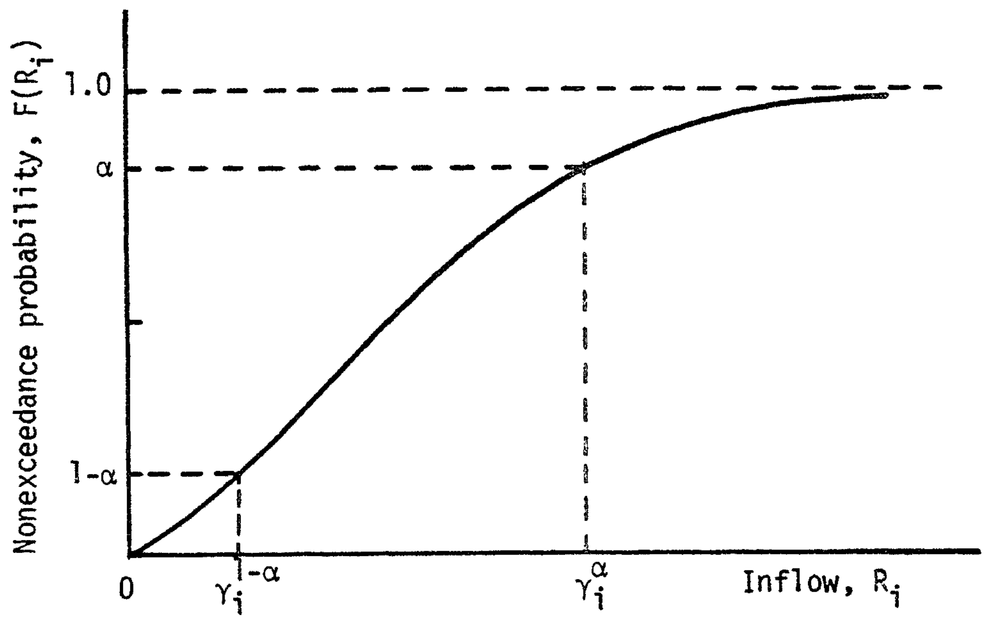


Figure 5-1 Distribution Function of the Random Inflow, R_i

that:

$$F^{\alpha}(R_i) = \text{Prob}\left\{ R_i \leq r_i^{\alpha} \right\} = \alpha \tag{5-11}$$

and,

$$F^{1-\alpha}(R_i) = \text{Prob}\left\{ R_i \leq r_i^{1-\alpha} \right\} = 1 - \alpha \tag{5-12}$$

Equation 5-12 can be expressed as :

$$F^{1-\alpha}(R_i) = \text{Prob}\left\{ R_i \geq \gamma_i^{1-\alpha} \right\} = \alpha \tag{5-13}$$

Using Equations 5-11 - 5-13, the deterministic equivalents of the chance-constraints (Equations 5-10) become:

$$C - v_i - b_i \geq r_i^{\alpha_1} \quad (i = 1, 2...12) \qquad \text{5-14 (a)}$$

$$m_i - b_i \leq r_i^{1-\alpha_2} \quad (i = 1, 2...12) \qquad \text{5-14 (b)}$$

$$q_i - b_{i-1} + b_i \leq r_{i-1}^{1-\alpha_3} \quad (i = 1, 2...12) \qquad \text{5-14 (c)}$$

$$f_i - b_{i-1} + b_i \geq r_i^{\alpha_4} \quad (i = 1, 2...12) \qquad \text{5-14 (d)}$$

where, r_i^{α} is the α-percentile flow value shown in Figure 5-1. Transposing known quantities to the right-hand side and assuming that the minimum storage, m_i, is some positive fraction, a_m, of the capacity C, the chance-constrained formulation takes the final form:

$$\text{Minimize } C \qquad \text{5-16 (a)}$$

subject to,

$$C - b_i \geq v_i + r_i^{\alpha_1} \quad (i = 1, 2...12) \qquad \text{5-16 (b)}$$

$$a_m C - b_i \leq r_i^{1-\alpha_2} \quad (i = 1, 2...12) \qquad \text{5-16 (c)}$$

$$b_{i-1} - b_i \geq q_i - r_{i-1}^{1-\alpha_3} \quad (i = 2, 3...12) \qquad \text{5-16 (d)}$$

$$b_{12} - b_1 \geq q_1 - r_{12}^{1-\alpha_3} \quad (i = 1) \qquad \text{5-16 (e)}$$

$$b_{i-1} - b_i \leq f_i - r_i^{\alpha}4 \quad (i = 2, 3...12) \qquad 5\text{-}16\ (f)$$

$$b_{12} - b_i \leq f_1 - r_1^{\alpha}4 \quad (i = 1) \qquad 5\text{-}16\ (g)$$

$$C \geq 0$$

$$b_i \text{ unrestricted } \quad (i = 1, 2...12)$$

Since most linear programming algorithms implicitly assume that all the variables in the problem are non-negative, the decision variables, b_i, must be expressed in terms of two non-negative variables. This is accomplished using the transformation:

$$b_i = h_i - g_i \quad (i = 1, 2...12) \qquad (5\text{-}17)$$

where h_i, $g_i \geq 0$

Loucks Formulation: For the parameter λ_i equal to 1 in every month, Equations 5-9 (b) and 5-9 (c) are no longer probabilistic, since they are not functions of the random variable R_i (Loucks, 1970). Also from Equation 5-5 it is observed that, for λ_i = 1, the storage at the end of period t equals the decision constant, b_i. Hence, any constraints on storage merely restrict the range of the decision constants, b_i, and cause these to be independent of the inflow probability distribution.

Using the procedure discussed in the previous section, the probabilistic constraints 5-9 (d) and 5-9 (e) can be converted to their deterministic equivalents. The full chance-constrained programming problem in this case is:

$$\text{Minimize } C \qquad \text{5-18 (a)}$$

subject to,

$$C - b_i \geq v_i \qquad (i = 1, 2\ldots12) \qquad \text{5-18 (b)}$$

$$a_m C - b_i \leq 0 \qquad (i = 1, 2\ldots12) \qquad \text{5-18 (c)}$$

$$b_{i-1} - b_i \geq q_i - r_i^{1-\alpha_3} \qquad (i = 2, 3\ldots12) \qquad \text{5-18 (d)}$$

$$b_{12} - b_1 \geq q_1 - r_1^{1-\alpha_3} \qquad \text{5-18 (e)}$$

$$b_{i-1} - b_i \leq f_i - r_i^{\alpha_4} \qquad (i = 2, 3\ldots12) \qquad \text{5-18 (f)}$$

$$b_{12} - b_1 \leq f_1 - r_1^{\alpha_4} \qquad \text{5-18 (g)}$$

$$C \geq 0$$

$$b_i \text{ unrestricted } (i = 1,2\ldots12)$$

Input Requirements: Values must be assigned to the minimum storage, m_i, the flood-control storage, v_i, the maximum and minimum releases, f_i and q_i, respectively, and the percentile flows, r_i^{α}, in Equations 5-16 and 5-18.

Since in this study the optimal releases are sought under the existing design at Hoover Reservoir, the fraction a_m is set at 0.0254, which is equal to the ratio of the minimum usable storage (2,188 acre-feet) to the existing reservoir capactiy (86,1228 acre-feet). The flood-control reservation, v_i, is 25,7808 acre-feet, the difference between the existing reservoir capacity (86,1228 acre-feet) and the storage below the crest of the spillway (60,342 acre-feet).

Values for the minimum and maximum releases are restricted by the feasibility requirement for their corresponding constraints. These restrictions can be found by summing the constraints on the minimum and maximum releases, respectively. This yields:

$$\sum_{i=1}^{12} q_i \leq \sum_{i=1}^{12} r_i^{1-\alpha_3} \tag{5-19}$$

and,

$$\sum_{i=1}^{12} f_i \geq r_i^{\alpha_4} \tag{5-20}$$

Table 5-1 lists percentile inflows, r_i^{α}, in each month at selected probability levels, α. These values are obtained from the fitted theoretical cumulative probability distribution of the historical flows observed at the reservoir site. (Refer to Appendix D-3)

Since the main purpose of Hoover reservoir is to meet water supply requirements, it is reasonable to set the minimum guaranteed flow, q_i, at the maximum value which satisfies the feasibility requirement in Equation 5-19. Assuming the same value of q_i in all months, Equation 5-19 gives:

$$q_i = \sum_{i=1}^{12} r_i^{1-\alpha_3} /12 \tag{5-21}$$

where values for $r_i^{1-\alpha_3}$ are shown in Table 5-1. Figure 5-2 displays the minimum flow, q_i, that can be guaranteed at different levels of reliability, α_3. It must be emphasized that the condition imposed on the minimum guaranteed flow by Equation 5-21 makes the constraints in Equations 5-16 (d) - 5-16 (e) and 5-18 (d) - 5-18 (e) binding. Thus, for this particular case, the decision constants, b_i, are determined entirely by the constraints on the minimum guaranteed flow, q_i.

Table 5-1: Percentile Inflows to Hoover Reservoir (cfs)

Month	Non-Exceedence Probability, α									
	0.98	0.95	0.90	0.80	0.70	0.30	0.20	0.10	0.05	0.02
January	3463.378	1808.043	1096.633	589.928	372.412	83.096	53.517	27.660	16.610	9.488
February	1669.034	1187.969	880.069	601.845	454.865	188.670	145.474	99.484	72.967	50.401
March	1274.106	972.627	772.784	589.928	482.992	244.692	200.337	148.413	119.104	90.017
April	1450.988	1032.770	749.945	518.013	403.429	164.022	125.211	85.627	63.434	44.701
May	804.322	544.572	379.935	249.635	183.094	66.686	49.402	31.817	22.198	15.029
June	1603.590	943.881	584.058	327.013	214.863	54.598	35.874	19.886	12.183	7.029
July	1118.787	601.845	336.972	168.174	104.585	19.688	12.183	6.050	3.456	1.822
August	323.759	160.774	85.627	39.646	22.646	3.669	2.117	0.980	0.538	0.267
September	101.494	53.517	29.371	14.154	8.331	1.492	0.905	0.440	0.242	0.123
October	94.632	54.598	33.116	18.541	12.183	2.945	1.916	1.067	0.657	0.387
November	1096.633	518.013	273.144	127.740	73.700	11.941	6.821	3.158	1.682	0.819
December	2697.281	1408.105	699.244	330.300	190.566	30.265	17.289	8.005	4.179	2.014

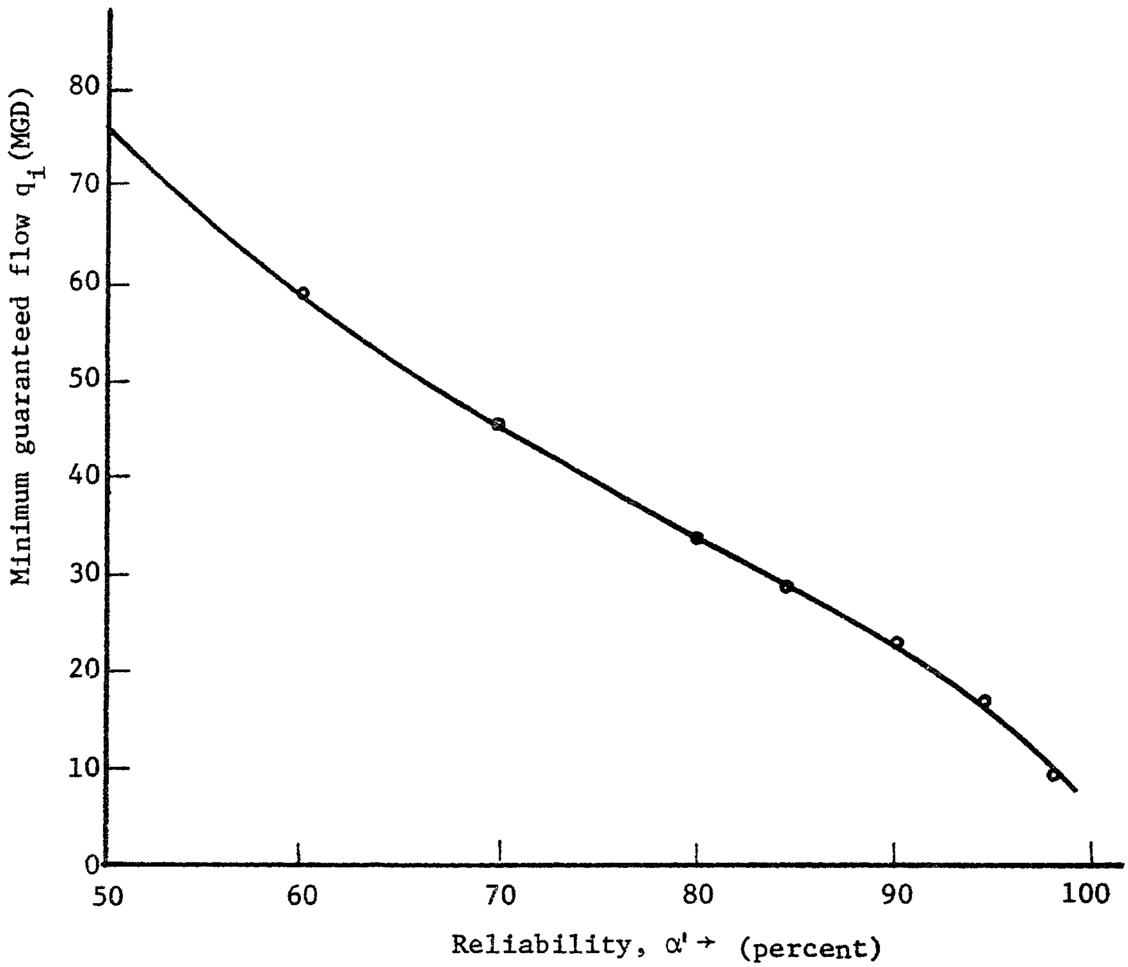


Figure 5-2: Minimum Guaranteed Flow, q_i, at Various Levels of Reliability, α'

The maximum allowable release, f_i, from Hoover Reservoir in any month is large enough to satisfy Equation 5-20 as a strict inequality. This implies that the constraints on the maximum release (Equations 5-16 (f) - 5-16 (g) and 5-18 (f) - 5-18 (g)) are not critical in the solution of the chance-constrained programming formulations.

<u>Dynamic Programming-Regression Method</u>: The monthly operation of a reservoir over a specified period may be conceived as a sequential decision process. In each month a decision must be taken as to the amount of water to be released from the reservoir under certain input and storage conditions. The release in any month depends on past releases and also affects future decisions. Reservoir releases in a particular month must be based on some criterion, generally taken to be economic in nature. Therefore, if an optimal set of releases are sought over the entire period of operation of the reservoir, the following mathematical problem applies:

$$\underset{(X\ ,\ X\ \ldots X_N)}{\text{Minimize}} \sum_{i=1}^{N} L(X_i) \tag{5-22}$$

subject to,

$$S_{i-1} = S_i + R_i - X_i \quad (i = 1, 2 \ldots N) \tag{5-23}$$
$$S_N = S_N^*$$
$$S_o = S_o^*$$

where,
- $L(X_i)$ = economic return function;
- X_i = release in month i;
- R_i = inflow in month i;
- S_{i-1} = storage at the end of month i;
- S_i = storage at the beginning of month i, and;
- S_N^*, S_o^* = initial and final storage levels in the reservoir, respectively.

In the above formulation index N represents the first month in the operation of the reservoir. This is done to maintain conformity with the dynamic programming formulation discussed later. The constraints in Equations 5-23 represent continuity requirements.

A solution to the above problem can be obtained using the principles of dynamic programming. The monthly operation of the reservoir is represented by stages as shown in Figure 5-3. At each stage a sub-optimization problem is solved while incorporating the optimal returns from the previous stage. The solution is expressed in terms of the state of the system. For the reservoir operation problem, the application of the dynamic programming methodology requires the following definitions:

a) Stage, i: A stage is represented by each month in the reservoir operation;

b) Decision Variable, X_i: At every stage the decision variable is the amount of water to be released from the reservoir;

c) State Variable, S_i: The reservoir storage at the beginning of each month is assumed to be the state variable;

d) Transformation Function, $t_i(S_i, X_i)$: The continuity equation is used as a transformation function to relate the inputs and outputs at any stage i. Thus, the function t_i is given by:

$$S_{i-1} = t_i(S_i, X_i) = S_i + R_i - X_i$$

e) Number of Stages, N: This is assumed to be equal to the design life of the reservoir. In the present study N is taken to be 50 years or 600 months;

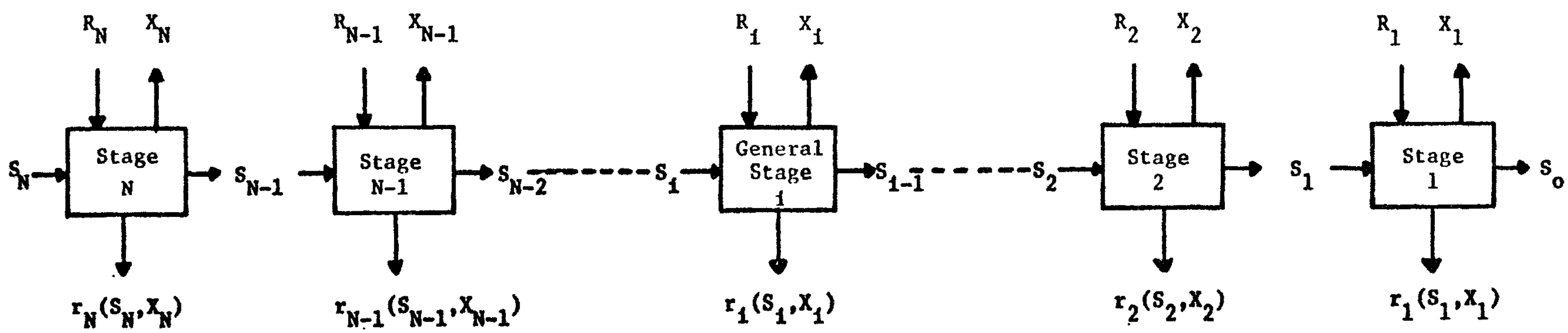


Figure 5-3 Multi-Stage Representation of Monthly Reservoir Operation.

f) <u>Return Function</u>, $r_i(S_i, X_i)$: At every stage this gives a measure of the economic value of releasing an amount, X_i, of water from the reservoir; and

g) <u>Optimal Return</u>, $f_i(S_i)$: This is the optimal return associated with the optimal release at stage i when the system is state S_i.

The problem stated in Equations 5-22 and 5-23 can now be solved using a discrete, backward-recursion dynamic programming algorithm. The first stage in the analysis represents the last month of reservoir operation and the solution proceeds backward in time. Thus the numbering of the stages is as shown in Figure 5-1. Such a procedure is often convenient from a computational point of view. The general recursive relationship for obtaining the optimal releases, given the terminal storages in the reservoir, is developed as follows:

<u>Stage 1</u>: For a given storage level (state variable) the return function $L_1(S_1, X_1)$ is optimized over all the feasible releases that yield an end storage equal to S_0. Thus,

$$f_1(S_1) = \underset{S_1 + R_1 - S_{max} \leq X_1 \leq S_1 + R_1 - S_o}{\text{minimize}} \; L_1(S_1, X_1) \tag{5-24}$$

subject to,

$$S_o = S_1 + R_1 - X_1$$

where,

S_{max} = storage below the flood pool.

Stage 2: At this stage the objective is to obtain the optimal release, X_2, that would result in an end storage S_1, given the beginning storage S_2. Hence,

$$f_2(S_2) = \underset{S_2 + R_2 - S_{max} \leq X_2 \leq S_2 + R_2 - S_{min}}{\text{minimize}} \left[L_2(S_2, X_2) + f_1(S_1) \right] \tag{5-25}$$

subject to,

$$S_1 = S_2 + R_2 - X_2$$

where,

S_{min} = minimum allowable storage.

Proceeding backward, the general recursive relationship for stage i is written as

$$f_i(S_i) = \underset{S_i + R_i - S_{max} \leq X_i \leq S_i + R_i - S_{min}}{\text{minimize}} \left[L_i(S_i, X_i) + f_{i-1}(S_{i-1}) \right] \tag{5-26}$$

subject to,

$$S_{i-1} = S_i + R_i - X_i \qquad (i = 1, 2 \ldots N)$$

and

$$S_N = S^*$$

$$S_o = S_o^*$$

After solving Equation 5-26 for all stages, the optimal releases for a given initial storage, S_N, can be obtained by retracing the optimal path, (i.e., from stage N to stage 1).

Constraints on the Release, X_i: At any stage, the range of releases defined by the constraints in Equation 5-26, may not everywhere be feasible. Therefore, to reduce the computations at any stage, only the feasible releases under the following conditions need be considered:

a) If $$(S_i + R_i) \leq S_{min};\ X_i = 0 \tag{5-27}$$

b) If $$S_{min} \leq (S_i + R_i) \leq S_{max};\ 0 \leq X_i \leq S_i + R_i - S_{min} \tag{5-28}$$

and,

c) If $$S_i + R_i \geq S_{max};\ S_i + R_i - S_{max} \leq X_i \leq S_i + R_i - S_{min} \tag{5-29}$$

The above conditions imply that if X_i is defined over the range (X_{min}, X_{max}) then,

$$X_{min} = \max\left\{ (S_i + R_i) - S_{max},\ 0 \right\}$$

and

$$X_{max} = \min\left\{ (S_i + R_i) - S_{min} \right\}$$

$$= S_i + R_i - S_{min} \tag{5-30}$$

Computational Considerations: At every stage, the following quantities are to be stored in the computer:

1) The optimal returns, $f_{i-1}(S_{i-1})$ from the previous stage for all the discrete storage levels, S_{i-1}. This information is required to solve the recursive relationship 5-26;

2) The optimal returns, $f_i(S_i)$ and the optimal set of releases $X_i^*(S_i)$ for all discrete storage levels, S_i;

The amount of stored information can be obtained using the formula

$$Q_T = 4N(t_1 + 2)t_2^P \tag{5-31}$$

where,

Q_T = amount of information in bytes;

N = number of stages;

t_1 = number of decision variables;

t_2 = number of discrete values of the state variable;

P = number of state variables.

In the present study, the usable storage in Hoover Reservoir (58,154 acre-feet) was discretised using 59 grid points at each stage. The maximum number of grid points for the release variable is equal to the grid size on the storage variable, since, for a given beginning storage S_i and inflow R_i, the release, X_i, is adjusted so as to correspond to one of the grid points of the end storage, S_{i-1}. Finally, the number of stages, N, in the problem is 600, the length of the operating period of the reservoir, in months. Under these conditions the computer storage required is 4(600)(1 + 2)(59) = 424,800 bytes = 424·8 K.

The solution to the recursive relationship in Equation 5-26 was programmed on an IBM 370/168 computer series, and used 35.80 seconds for execution time. The total computer storage, including all computations, was close to 632 K. A listing of the program is provided in Appendix I.

Regression Analysis: Optimal releases derived using the dynamic program can be regressed on important independent variables to derive monthly reservoir operating rules. Linear and non-linear functions of reservoir storage at the beginning of each period, current inflow and inflows in previous periods, are considered as representative release policies to fit the regression data. These policies are developed and analyzed in Chapter 7.

CHAPTER VI

Chance-Constrained Linear Programming Results

Solutions to the chance-constrained linear programming problems formulated in Chapter 5 are presented with Hoover Reservoir as a case example. Results highlighting some important characteristics of the chance-constrained linear programming approach, are illustrated. Monthly linear release policies are derived and tested in a simulation environment to examine their performance.

Optimal Policies at Various Reliability Levels

To determine the optimal operation of Hoover Reservoir under its existing capacity, the chance-constrained linear programs developed in Equations 5-16 and 5-18 must be solved at various levels of reliability, α. The purpose of such an analysis is to obtain the reliability levels which the physical and operational constraints must satisfy in order that the optimal reservoir capacity corresponds to the existing capacity at Hoover Reservoir. The following chance-constrained linear programming formulations are used in the analysis:

Model LDR1 (Revelle Formulation)

Release policy: $X_t = S_{t-1} - b_i$ (original form) (6-1)

or $X_t = R_{t-1} + b_{i-1} - b_i$ (transformed form)(6-2)

Linear program:

$$\text{Minimize } C \qquad 6\text{-}3\ (a)$$

subject to:

$$C - b_i \geq v_i + r_i^{\alpha^*} \quad (i = 1, 2 \ldots 12) \qquad 6\text{-}3\ (b)$$

$$a_m C - b_i \leq r_i^{1-\alpha^*} \quad (i = 1, 2 \ldots 12) \qquad 6\text{-}3\ (c)$$

$$b_{i-1} - b_i \geq q_i - r_{i-1}^{1-\alpha'} \quad (i = 1, 2 \ldots 12) \qquad 6\text{-}3\ (d)$$

$$b_{12} - b_1 \geq q_1 - r_{12}^{1-\alpha'} \qquad 6\text{-}3\ (e)$$

$$b_{i-1} - b_i \leq f_i - r_i^{\alpha^*} \quad (i = 2, 3 \ldots 12) \qquad 6\text{-}3\ (f)$$

$$b_{12} - b_1 \leq f_1 - r_1^{\alpha^*} \qquad 6\text{-}3\ (g)$$

$$C \geq 0$$

$$b_i \text{ unrestricted} \quad (i = 1, 2 \ldots 12)$$

Model LDR2 (Loucks Formulation)

Release policy: $X_t = S_{t-1} + R_t - b_i$ (original form) (6-4)

or $X_t = R_t + b_{i-1} - b_i$ (transformed form)(6-5)

Linear program:

$$\text{Minimize } C \qquad 6\text{-}6\ (a)$$

subject to:

$$C - b_i \geq v_i \quad (i = 1, 2 \ldots 12) \qquad 6\text{-}6\ (b)$$

$$a_m C - b_i \leq 0 \quad (i = 1, 2 \ldots 12) \qquad 6\text{-}6\ (c)$$

$$b_{i-1} - b_i \geq q_i - r_i^{1-\alpha'} \quad (i = 2, 3 \ldots 12) \qquad 6\text{-}6\ (d)$$

$$b_{12} - b_1 \geq q_1 - r_1^{1-\alpha'} \qquad 6\text{-}6\ (e)$$

$$b_{i-1} - b_i \leq f_i - r_i^{\alpha^*} \qquad (i = 2, 3 \ldots 12) \qquad 6\text{-}6\ (f)$$

$$b_{12} - b_1 \leq f_1 - r_1^{\alpha^*} \qquad 6\text{-}6\ (g)$$

$$C,\ b_i \geq 0$$

where, α^* = Reliability in satisfying the constraints on flood control storage, minimum storage, and maximum release requirements.

α' = Reliability associated with the minimum guaranteed flow.

Optimal monthly policies are derived by maintaining α^* constant while varying the reliability, α', on the minimum guaranteed flow, q_i. The solutions, presented in Appendix E , assume that the feasibility requirement of constraints 6-3 (d) - 6-3 (e) and 6-6 (d) - 6-6 (e) on minimum guaranteed flow are satisfied as strict equalities (see Equations 5-20 and 5-22), thereby making these constraints binding. As mentioned in Chapter 5, the constraints on maximum release, f_i, are not critical in the solution of the linear programming formulations. The above conditions imply that:

a) In general, the decision constants, b_i, for Model LDR1 depend upon the reliability levels α^* and α'; hence release policies in their original form (Equations 6-1 and 6-4) are dependent on the reliability with which all constraints are satisfied. Model LDR2 is a special case where release policies are independent of α^*. However, for the Hoover Reservoir case-study, the decision constants are not affected by the reliability imposed on the maximum release constraints since these always hold as strict inequalities under both models.

b) For the case study, the relative magnitude of the difference $b_{i-1} - b_i$ is determined only by the minimum guaranteed flow constraints. This is evident since, by virtue of the feasibility requirement discussed earlier (Equation 5-21), these constraints are always binding. Therefore, the values of $b_{i-1} - b_i$ depend on the reliability, α', imposed on the minimum guaranteed flow constraints, and are not affected by the reliability α^*.

c) Conclusion (b) may be useful if the transformed forms of the linear release policies (Equations 6-2 and 6-5) are adopted as the monthly release policies. Since such policies are functions of the quantity, $b_{i-1} - b_i$, they depend only on the reliability of meeting minimum guaranteed flow. In the last section of this Chapter, reservoir operating performance under the transformed linear release policies is compared with the original forms of these policies.

d) The transformed form of the release policy in any month under LDR1 is identical to the transformed form of the release policy in the previous month but derived using model LDR2. This can be illustrated by considering release policies in two consecutive months, t-1 and t, respectively. The transformed release policy under LDR1 for month t is:

$$X_t = R_{t-1} + (b_{i-1} - b_i)_{t,\ \mathrm{LDR1}}$$

while the policy in month t-1 and under LDR2 may be expressed as:

$$X_{t-1} = R_{t-1} + (b_{i-1} - b_i)_{t-1,\ LDR2}$$

Since $(b_{i-1} - b_i)_{t-1,\ LDR2}$ is equal to $(b_{i-1} - b_i)_{t,\ LDR1}$ (see Table 6-1), the above two policies are identical.

The identity of the difference in b values under the two models occurs since the constants, $b_{i-1} - b_i$, in the transformed release policies are determined by the minimum guaranteed flow constraints -- for which the right-hand side coefficients in any month i, under LDR1, are equal to the coefficients in month i-1 corresponding to model LDR2. (It is assumed that the minimum guaranteed flow, q_i, is the same in all months.)

The relationship between the optimal capacity, C, and a general reliability, α, imposed on all constraints in the chance-constrained models LDR1 and LDR2, is shown in Figures 6-1 (a) and 6-1 (b). These figures are useful in determining the maximum reliability with which all the operational constraints can be satisfied given that a reservoir already exists. They also reflect the trade-off between reservoir capacity and degree of reliability imposed on the constraints. For LDR1, capacity increases with an increase in the reliability, α, while the reverse is true for LDR2.

Figures 6-2 (a) and 6-2 (b) illustrate the nature of the dependence between optimal capacity and reliability, α', on the minimum guaranteed flow at various levels of reliability, α^*, imposed on the other constraints. Under both models it is seen that a decrease in reliability

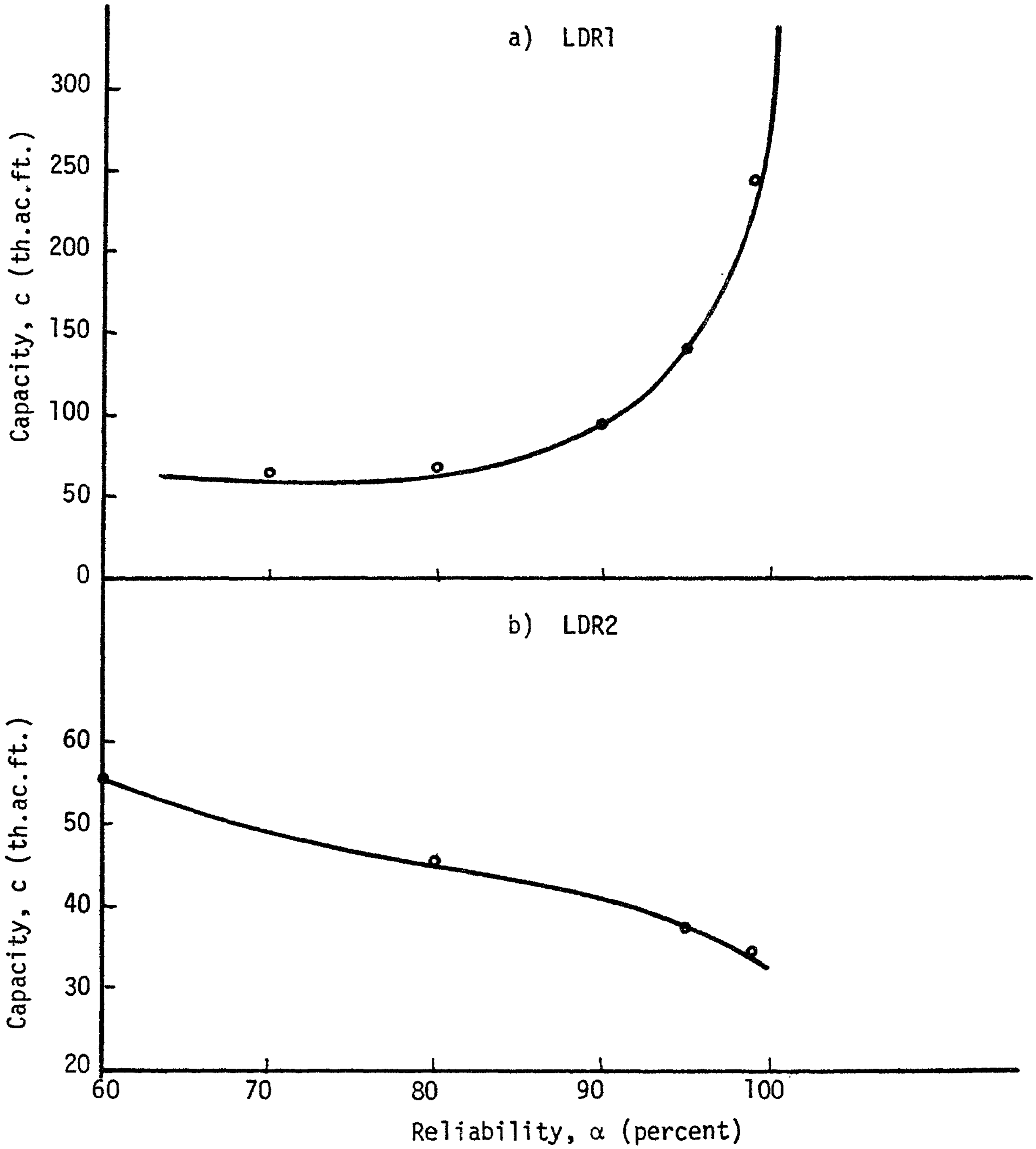


Figure 6-1: Required Reservoir Capacity at Various Levels of Reliability, α, Imposed on All Constraints

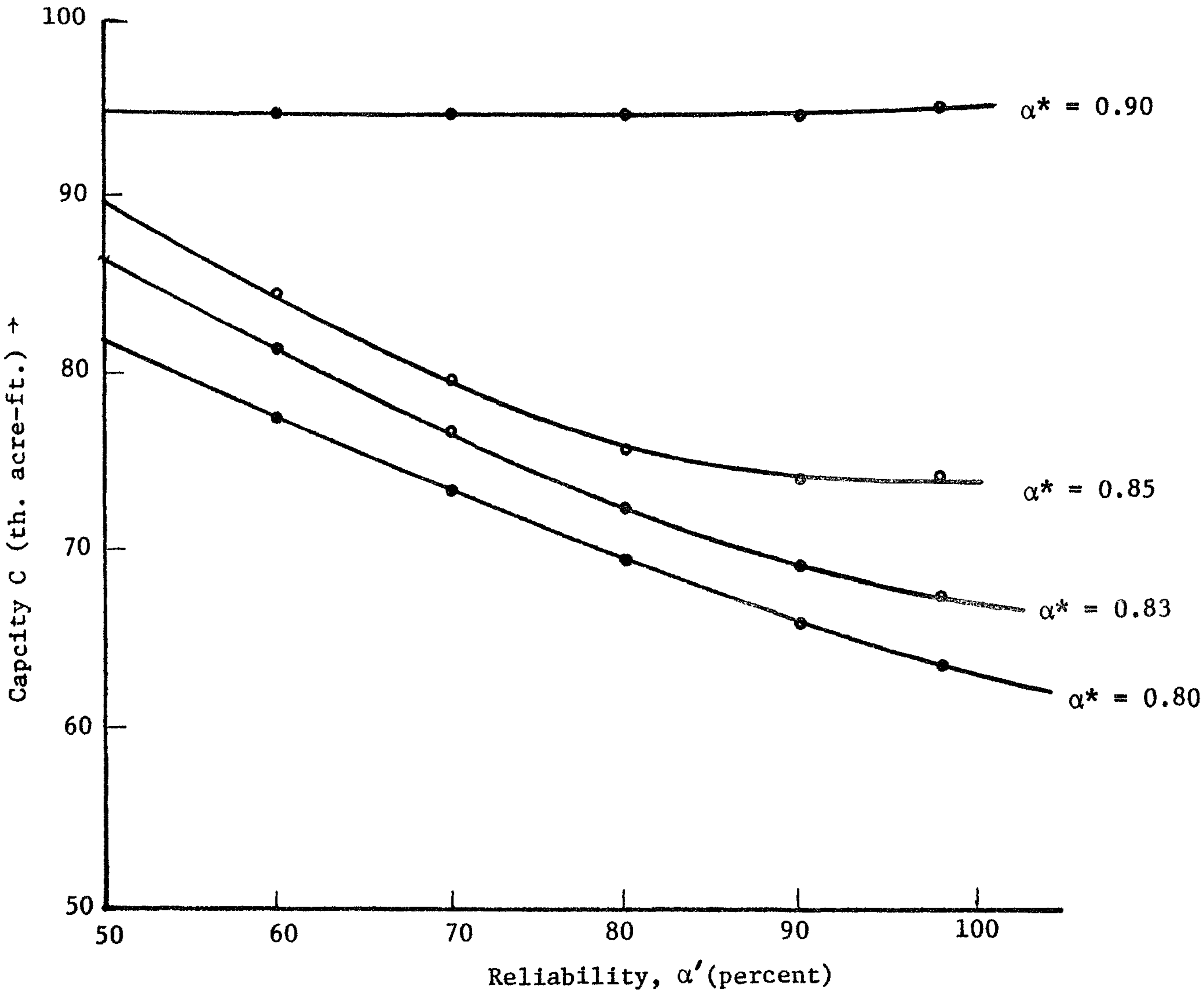


Figure 6-2(a): Required Reservoir Capacity, C, at Various Levels of Reliability on the Minimum Guaranteed Flow, q_i, Model LDR1

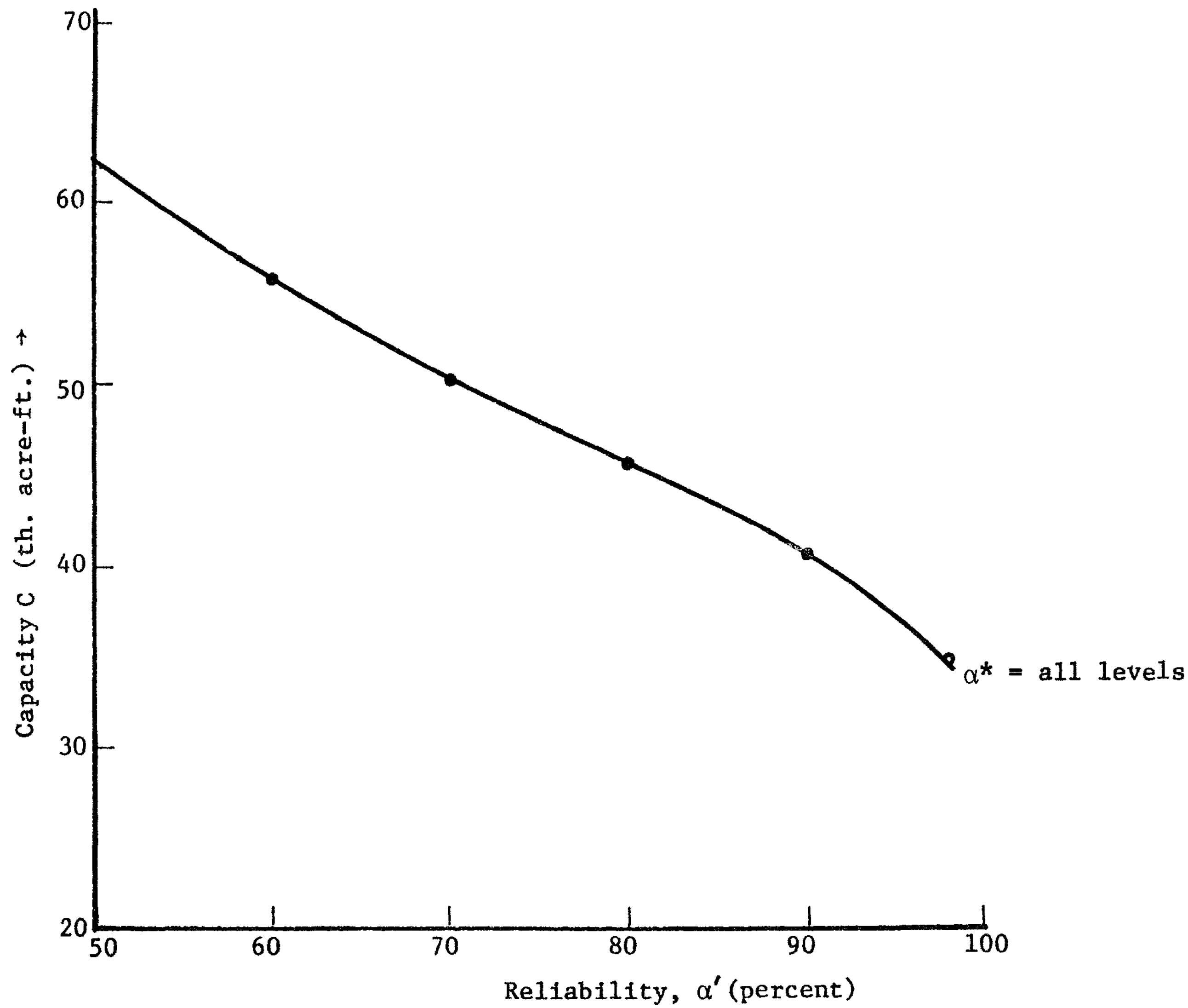


Figure 6-2(b): Required Reservoir Capacity, c, at Various Levels of Reliability on the Minimum Guaranteed Flow, q_i, Model LDR2 on the minimum guaranteed flow requires a larger reservoir capacity. For LDR1 the level of reliability, α^*, will affect the capacity through the freeboard and minimum storage constraints (Equations 6-3 (b) and 6-3 (c)), in general. Consequently, as Figure 6-2 (a) demonstrates, the relationship between capacity and the reliability, α', depends on the level of α^*. In the case of LDR2 the reliability level, α^*, does not appear in either the freeboard or minimum storage constraints. Thus, it is expected that capacity will not vary with the choice of α^* in the model.

Optimal Monthly Release Policies

The information provided in Figures 6-2 (a) and 6-2 (b) is used to determine the reliability levels which the constraints in Equations 6-3 and 6-6 must satisfy, in order that the optimal capacity corresponds to the existing capacity of Hoover Reservoir. However, the reliability, α', on the minimum guaranteed flow constraints is independently set, using Figure 5-2, at a level of 0.50. This ensures a minimum guaranteed flow of 75.0 M.G.D., the current draft rate from Hoover Reservoir. With α' at this level and the existing Hoover Reservoir capacity (86.1228 th. ac. ft.), the reliability α^* from Figure 6-2 (a) is 0.83 for model LDR1. However, for model LDR2, there is no control over the reliability α^*. It is also observed from Figure 6-2 (b) that in this case the optimal capacity required to guarantee the target draft of 75.0 M.G.D. (with a 0.50 reliability) is less than the existing Hoover Reservoir capacity. Table 6-1 presents the optimal monthly release policies, derived for LDR1 and LDR2 under the above operating conditions.

It is shown in Appendix E that these optimal release policies remain unchanged when a two-sided quadratic loss function is substituted as the objective function under the same sets of constraints as above for the chance-constrained models. The objective function is:

$$\text{Minimize } Z = \sum_{t=1}^{N} E\,(X_t - T_t)^2$$

where,

T_t = target release.

The same operating conditions as above are also maintained in this formulation.

Simulation Results

The operation of Hoover Reservoir is simulated using the monthly release policies presented in Table 6-1. The simulations are carried out using 20 synthetically generated inflow sequences, each of 148 years duration. The objective of such an analysis is to examine the overall performance of the optimal release policies derived using the chance-constrained linear programming models. Averages of important performance measures, based on simulation runs at a target of 75.0 M.G.D. are illustrated in Tables 6-2 - 6-6. Similar data at a target level of 12.0 M.G.D. are shown in Appendix F. Simulation results for the original and transformed release policies are included in these Tables. For model LDR2, the optimal capacity is below the existing Hoover Reservoir capacity. Simulation results under this optimal capacity indicate that, at a target level of 75.0 M.G.D., the performance characteristics do not change significantly when the reservoir is simulated using the transformed release policies and the existing

Table 6-1: Optimal Monthly Release Policies
(All units in thousand acre-feet)

Month, i	Model LDR1[1]		Model LDR2[2]	
	b_i	$b_{i-1} - b_i$	b_i	$b_{i-1} - b_i$
January	-0.5780	2.5772	5.7351	-3.4980
February	2.9179	-3.4960	14.7825	-9.0474
March	11.9668	-9.0489	28.2736	-13.4911
April	25.4579	-13.4911	36.3613	-8.0877
May	33.5456	-8.0877	35.9176	0.4437
June	33.1019	0.4437	35.1269	0.7907
July	32.3112	0.7907	30.7060	4.4209
August	27.8903	4.4209	24.0677	6.6383
September	21.2520	6.6383	17.0828	6.9849
October	14.2671	6.9849	10.2619	6.8209
November	7.4462	6.8209	4.8143	5.4476
December	1.9986	5.4476	2.2371	2.5772

1 C = 86.1228 th. ac. ft.
v_i = 25.7808 th. ac. ft.
S_{min} = 2.188 th. ac. ft.
q_i = 75.0 M.G.D.
α' = 0.50
α^* = 0.83

2 C = 62.1421 th. ac. ft.
v_i = 25.7808 th. ac. ft.
S_{min} = 2.188 th. ac. ft.
q_i = 75.0 M.G.D.
α' = 0.50
α^* = uncontrolled

Table 6-2: Statistics on Optimal Monthly Releases
(all units in thousand acre-feet, 75 .0 M.G.D. target)*

	Model LDR1				Model LDR2			
	Original Policy ($X_t = S_t - b_i$)		Transformed Policy ($X_t = R_{t-1} + b_{i-1} - b_i$)		Original Policy ($X_t = S_{t-1} + R_t - b_i$)		Transformed Policy ($X_t = R_t + b_{i-1} - b_i$)	
Month	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
January	15.922	19.042	15.464	19.172	18.452	23.566	18.452	23.566
February	16.678	16.016	17.581	19.043	11.892	11.402	11.916	11.386
March	12.557	12.780	12.386	12.524	11.534	13.414	11.605	13.386
April	11.825	13.028	12.240	14.319	11.170	11.707	11.631	11.541
May	10.700	9.505	11.892	11.566	11.317	9.309	11.732	9.208
June	11.879	11.011	12.317	11.261	12.707	11.738	12.752	11.707
July	11.645	8.176	12.629	11.265	10.213	5.843	10.225	5.829
August	10.151	5.423	9.950	5.708	9.515	4.810	9.515	4.810
September	9.458	4.417	8.995	4.913	8.231	2.085	8.229	2.087
October	8.231	2.085	7.482	2.852	7.499	0.879	7.457	0.962
November	7.499	0.879	6.631	1.989	9.702	4.152	9.313	4.306
December	10.144	6.496	9.198	6.592	14.462	13.233	13.867	13.256
TOTAL	136.689		136.765		136.694		136.694	

1 acre-foot = cubic meters; 75 .0 M.G.D. = acre-feet per month

Table 6-3: Statistics on Beginning Monthly Storage
(all units in thousand acre-feet, 75.0 M.G.D. target)

	Model LDR1				Model LDR2			
	Original Policy ($X_t = S_t - b_i$)		Transformed Policy ($X_t = R_{t-1} + b_{i-1} - b_i$)		Original Policy ($X_t = S_{t-1} + R_t - b_i$)		Transformed Policy ($X_t = R_t + b_{i-1} - b_i$)	
Month	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
January	13.436	11.104	9.043	10.734	2.237	0.185	2.237	0.185
February	19.419	15.789	15.483	15.463	5.689	0.530	5.689	0.529
March	23.545	11.107	18.707	10.982	14.602	1.395	14.578	1.403
April	35.391	12.147	30.724	12.761	27.471	2.987	27.376	2.998
May	43.153	9.924	38.072	11.262	35.888	3.393	35.333	3.632
June	43.742	8.091	37.469	10.409	35.859	3.014	34.889	3.605
July	43.824	8.552	37.113	10.851	35.114	2.910	34.098	3.550
August	37.983	6.026	30.289	8.661	30.705	2.533	29.677	3.264
September	30.710	5.047	23.216	7.942	24.068	1.985	23.039	2.875
October	22.498	2.716	15.468	5.889	17.083	1.409	16.057	2.514
November	14.946	1.383	8.664	4.049	10.262	0.847	9.279	2.098
December	11.700	4.188	6.286	4.667	4.814	0.398	4.220	1.052

Table 6-4: Statistics on Monthly Shortages
(Percent Violation, 75.0 M.G.D. target)*

Month	Model LDR1 Original Policy	Model LDR1 Transformed Policy	Model LDR2 Original Policy	Model LDR2 Transformed Policy
January	27.03	30.78	35.27	35.27
February	33.11	33.14	38.72	38.58
March	41.05	41.12	49.16	48.78
April	47.67	47.50	44.80	41.96
May	44.22	42.37	38.41	35.88
June	36.96	34.60	35.10	34.87
July	37.03	37.13	30.95	30.91
August	30.95	32.80	21.01	21.01
September	9.80	15.51	0.00	0.10
October	12.16	23.55	28.28	29.53
November	1.12	25.74	18.45	24.70
December	23.21	34.73	26.89	31.35

*equivalent to minimum guaranteed flow constraint violations, $\alpha' = 50$ percent.

Table 6-5: Statistics on Average Monthly Loss
(75.0 M.G.D. target)

Month	Model LDR1 Original Policy	Model LDR1 Transformed Policy	Model LDR2 Original Policy	Model LDR2 Transformed Policy
January	466.379	466.302	698.175	698.175
February	359.969	487.951	158.726	158.625
March	192.411	184.471	200.389	200.283
April	194.735	235.335	156.971	157.155
May	103.015	157.116	104.330	106.112
June	151.800	160.371	176.988	176.776
July	86.880	158.785	44.502	44.416
August	38.231	40.776	29.683	29.683
September	26.061	29.160	5.771	5.769
October	5.212	7.974	0.528	0.659
November	0.750	3.742	24.946	24.241
December	61.843	55.177	241.138	233.518
TOTAL	1687.286	1987.160	1842.147	1835.412

Table 6-6: Monthly Performance of Chance-Constrained Models (75.0 M.G.D. target)

Month	Model LDR1 Original Policy	Model LDR1 Transformed Policy	Model LDR2 Original Policy	Model LDR2 Transformed Policy
I Negative Predicted Releases (percent)				
January	0.00	0.00	5.17	5.17
February	5.17	4.29	9.73	9.22
March	9.46	7.79	22.36	21.35
April	22.36	18.39	13.89	8.11
May	13.89	6.86	2.06	0.00
June	2.03	0.00	0.41	0.00
July	0.41	0.00	0.03	0.00
August	0.03	0.00	0.00	0.00
September	0.00	0.00	0.00	0.00
October	0.00	0.00	0.00	0.00
November	0.00	0.00	0.00	0.00
December	0.00	0.00	0.00	0.00
II Month End Maximum Storage Violations (percent)				
January	5.10	3.12	0.00	0.00
February	1.35	0.49	0.00	0.00
March	6.22	2.94	0.00	0.00
April	8.99	5.01	0.00	17.70
May	5.47	2.36	0.00	0.00
June	8.14	3.18	0.00	0.00
July	1.15	0.14	0.00	0.00
August	0.57	0.05	0.00	0.00
September	0.00	0.00	0.00	0.00
October	0.00	0.00	0.00	0.00
November	0.00	0.00	0.00	0.00
December	1.69	0.62	0.00	0.00
III Month End Minimum Storage Violations (percent)				
January	2.74	0.00	0.00	0.00
February	0.00	0.00	0.00	0.00
March	0.00	0.00	0.00	0.00
April	0.00	0.00	0.00	0.00
May	0.00	0.00	0.00	0.00
June	0.00	0.00	0.00	0.00
July	0.00	0.00	0.00	0.00
August	0.00	0.00	0.00	0.00
September	0.00	0.00	0.00	0.10
October	0.00	0.00	0.00	2.26
November	0.00	0.00	0.00	15.61
December	0.00	0.00	100.00	99.93

Hoover Reservoir capacity of 86.1228 th. ac. ft. For example, the average loss per year, as shown in Table 6-7, increases by 0.08% when the existing design is used instead of the optimal design. Under the original release policy, the performance is not affected when the reservoir capacity is increased.

At targets of 60.0 and 12.0 M.G.D., as presented in Table 6-7, the optimal reservoir capacity is below the existing Hoover Reservoir capacity for both models LDR1 and LDR2. By assuming the existing Hoover Reservoir capacity, however, the maximum change in the average loss per year is only 2% (see Table 6-7). Higher target levels, above 75.0 M.G.D., were not considered in the analysis since the solution under the chance-constrained formulation would not be meaningful due to further reduction in the reliability, α', on minimum guaranteed flow (refer to Figures 6-2 (a) - (b) and Figure 5-2).

Table 6-8 demonstrates the average annual loss relative to the standard policy at different target levels. These results are based on the existing Hoover Reservoir design.

For the original linear decision rule policies, the simulation results presented in Tables 6-2 - 6-6, Table 6-8 and Appendix F demonstrate that:

a) The simulation estimates of reliability in satisfying the chance-constraints on the maximum storage, minimum storage and minimum release are well within the selected levels of reliability. It is also observed that, although the level of reliabilities were set equal in all months, simulation indicates a varying degree of reliability between months. In most cases

Table 6-7: Chance-Constrained Model Performance at Various Target Levels

Reliability Levels	Target (MGD)	Existing Design				Optimal Design			
		LDR1		LDR2		LDR1		LDR2	
		Original	Transformed	Original	Transformed	Original	Transformed	Original	Transformed
$\alpha^* = 0.83$ $\alpha' = 0.50$	75	1686.985	1987.160	1841.827	1836.608	1686.985	1987.160	1841.827	1835.093
$\alpha^* = 0.83$ $\alpha' = 0.60$	60	1936.271	2216.815	2099.027	2094.960	1929.948	2243.596	2096.961	2092.478
$\alpha^* = 0.83$ $\alpha' = 0.98$	12	3232.726	3450.467	3398.518	3395.220	3197.165	3525.190	3395.193	3395.177

Table 6-8: Comparison of Average Annual Loss

Release Policy	Average Loss/Year Target (M.G.D.)			Percentage Change in Loss Over Standard Policy Target (M.G.D.)		
	75.0	60.0	12.0	75.0	60.0	12.0
1. Standard Policy	1786.791	2240.273	3603.777	---	---	---
2. LDR1 (original)	1687.286	1936.271	3232.726	-5.57	-13.57	-10.30
3. LDR1 (transformed)	1987.160	2216.815	3450.467	11.21	-1.05	-4.25
4. LDR2 (original)	1842.147	2099.027	3398.518	3.10	-6.31	-5.70
5. LDR2 (transformed)	1835.412	2094.960	3395.220	2.72	-6.49	-5.79

the variability is intuitive; e.g., it is physically reasonable to expect that the probability of exceeding the maximum storage, S_{max}, would be greater in the months of high flows, January-April, as compared to the low flow months, August-November. In some cases the deviations are not as intuitive; one would anticipate that, to minimize total squared deviations of releases from the target that losses would be rather evenly distributed among the months (to avoid heavy penalties for large deviations in any month). Simulation indicates, however, that losses are highest for the high flow months January-June, and are very small for the low flow months September-November. It therefore seems that the adjustment toward uniformity is restricted by the physical constraints imposed by a finite reservoir capacity.

b) The average storage at the end of any month, i, is equal to the decision constant b_i for model LDR2 (refer to Table 6-3). This is expected since substitution of the original release policy into the continuity equation,

$$S_t = S_{t-1} + R_t - X_t$$

yields

$$S_t = S_{t-1} + R_t - S_{t-1} - R_t + b_i$$

or,

$$S_t = b_i \qquad (6\text{-}7)$$

On the other hand, for model LDR1 the storage at the end of the month, S_t, is random since it depends on the inflow R_t. In this case substitution of the original release policy into the continuity equation gives

$$S_t = S_{t-1} + R_t - S_{t-1} + b_i$$

or,

$$S_t = R_t + b_i \tag{6-8}$$

c) Statistics on the average loss per year, presented in Table 6-8, indicate that optimal policies derived using the chance constrained linear programming approach yield losses that are very similar to those incurred under the standard policy. For instance, at a target of 75.0 M.G.D., policies under LDR1 (original form) show a nominal decrease of 5.57% in the average loss per year over the standard policy. In contrast, policies obtained from LDR2 (original form) exhibit an increase of 3.10% in the average loss per year. However, statistics on the reliability of meeting similar monthly draft levels shows that the standard policy gives better reliability levels than the chance-constrained programming release policies at a target of 75 M.G.D.

d) The suitability of the chance-constrained approach to derive a release policy for a particular reservoir site is restricted by the relationship between the minimum guaranteed flow, q_{min}, and the corresponding reliability, α', imposed on this constraint. Figure 5-2 shows that for the case-study under consideration, the reliability of meeting the safe-yield of 68 M.G.D. (or equivalently, 75 M.G.D. including evaporation) is about 55%, while simulation under a standard policy indicates a reliability of 98% (refer to Table 4.7). This suggests that the chance-constrained programming approach is not adequate to provide reasonable monthly policies, particularly when the minimum guaranteed flow, q_{min}, is set equal to the safe-yield of the reservoir. The reliability of meeting such drafts as projected by the chance-constrained programming approach is too conservative. The standard policy is also better at any target yield below 68 M.G.D., since the corresponding reliability is very high (greater than 0.98). At targets above 68 M.G.D. performance of the standard policy is superior since the reliability levels in the chance-constrained models become very low for large values of q_{min}, while reliabilities under the standard policy do not deteriorate as rapidly.

Comparison of Original and Transformed Policies

Results presented in Tables 6-2 - 6-8 and Appendix F suggest that, for model LDR1, operational differences exist between the original and transformed release policies. In contrast, the two forms of release policies are similar under model LDR2. It may be recalled from Chapter V that the transformed release policy is merely a transformation of the original policy using the continuity equation. Thus, it is expected that the predicted releases would be identical under these two forms of policies, provided that continuity in reservoir storage is maintained between each period of operation. However, for a reservoir of finite capacity, the continuity equation is not satisfied if:

a) the maximum and minimum limits of the usable storage (S_{max} and S_{min}, respectively) are violated.

b) predicted releases are negative, in which case these are assigned a zero value in the present simulation study.

The frequency of occurrence* of the above two conditions in a simulation experiment determines the extent of the discrepancy between the transformed and original release policies. For model LDR1, both conditions a) and b) would be observed, since the end period storage, S_t, is random (see Equation 6-8). While for model LDR2 the only discontinuities would arise due to condition b). Maximum and minimum storage violations would not be expected (except those caused by negative releases) since the end period storage, S_t, is equal to the constant, b_i (see Equation 6-7). Also, for model LDR2 the maximum and minimum values of b_i are identical to the storage limits S_{max} and S_{min}, respectively (refer to Table 6-1).

From the above discussion it is recommended that the linear release policies, LDR1 and LDR2, be used in their original form to avoid discrepancies caused by operational discontinuities and to achieve the lower average losses associated with this form.

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*This depends on the reliability levels α^* and α', imposed on the chance constraints (Equations 6-3 and 6-6).

Chapter VII

DYNAMIC PROGRAMMING - REGRESSION RESULTS

A single multi-purpose reservoir is analyzed using a backward-recursion dynamic programming algorithm to obtain optimal releases. The dynamic program is solved for both one-sided and two-sided quadratic loss functions. In the first section monthly policies are derived by regressing the optimal set of releases on the input and state variables. Linear and nonlinear release policies are developed, then verified and compared through simulation. It is also illustrated that the maximum R^2 criterion for selecting release policies may not always be appropriate. As a special case, the relative performance of these policies, derived under a two-sided loss function, is compared at different levels of reservoir mean detention time.* The last section is devoted to verification of the dynamic program algorithm by comparing annual release policies with similar work by Young (1966). Hoover Reservoir is used as a case example.

Derivation of Monthly Policies: The dynamic programming recursive relationship (Equation 5-26) is solved using a 50-year (600-month)

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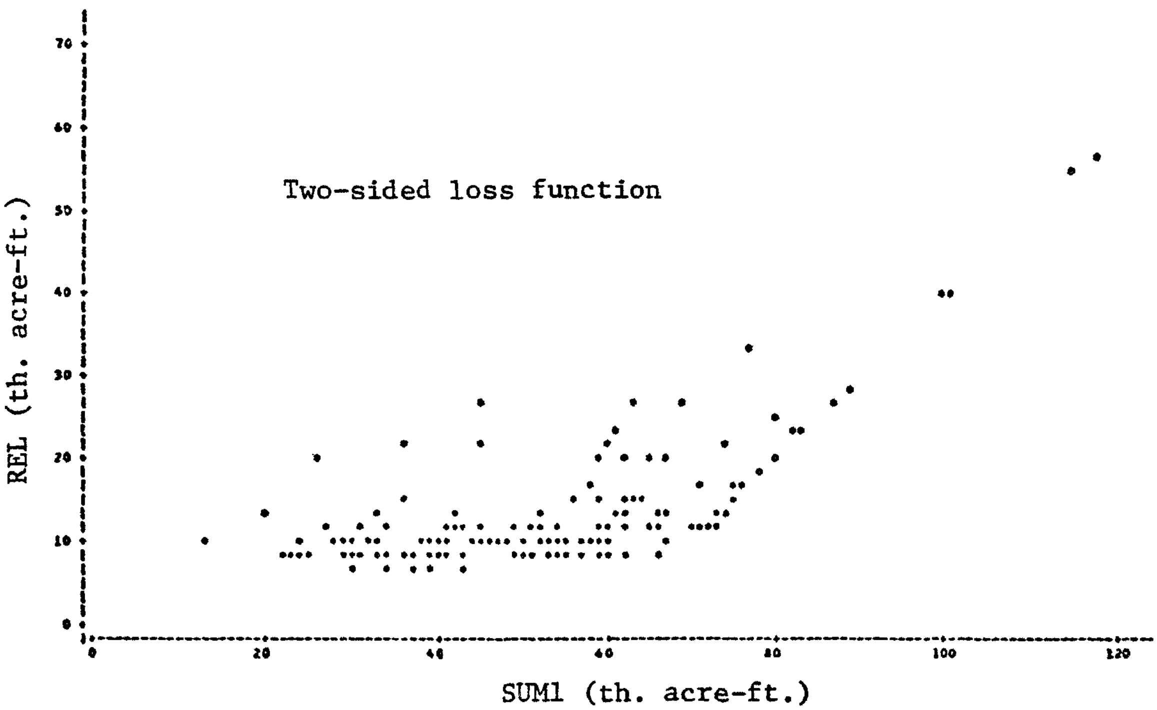
*This is defined as the ratio of maximum usable storage, SMAX, to the mean annual inflow, μ_x, i.e., MDT = SMAX/μ_x

generated inflow sequence. Both two-sided and one-sided quadratic loss functions are used to define losses, $l_t(X_t)$ as a function of release, X_t, in any month t. For the one-sided loss function, releases in excess of the target-release are associated with a zero loss. Such a loss function is appropriate whenever releases greater than the target have no economic value.

The reservoir is assumed to be full at the beginning and empty at the end of operation. Thus, S_N^* and S_0^* in Equation 5-26 are set at the SMAX and SMIN, respectively. Observations from the initial and final period of operation are omitted, since for Hoover Reservoir the initial and terminal storage conditions only affect the optimal releases in the first and last years of operation, respectively. For purposes of accuracy in the regression analysis, results of three independent solutions of the dynamic programming recursive relationship with different inflow squences are combined.

Figure 7.1 demonstrates a scatter plot of optimal releases in relation to reservoir content, SUM1 = (inflow + beginning period storage), for a representative high flow month. Similar scatter plots, illustrating the nature of the relationship between optimal releases and other important variables, are included in Appendix G.

The scatter diagram in Figure 7.1 suggests that for a two-sided quadratic loss function and a target of 75 MGD, the optimal releases derived under the dynamic program show considerable departure from the standard policy. For a one-sided



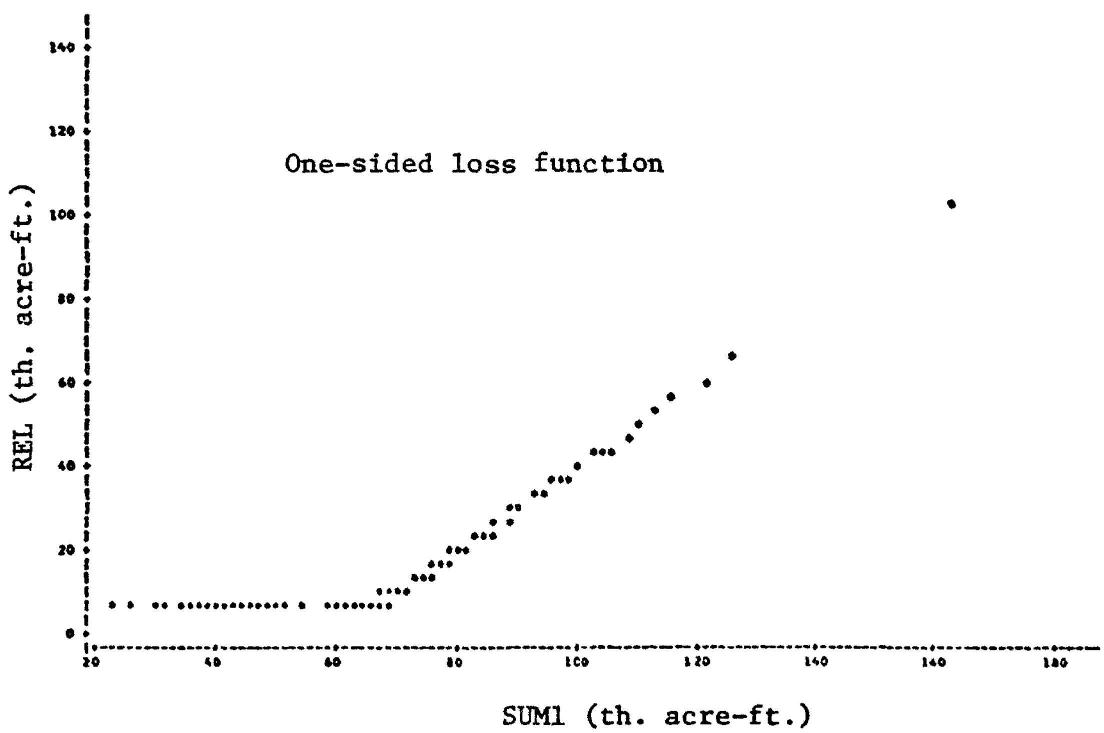


Figure 7-1: Scatter Plot of Optimal Releases at Different Levels of (Inflow + Storage) in March (Target = 75 MGD)

loss function, the optimal releases are close to the standard policy.

Simple correlation coefficients between optimal monthly releases and certain independent variables are given in Table 7.1 for a target level of 75 MGD. The following definitions are used in the table:

QFL = inflow in month i

QFL1, QFL2,...QFL4 = lagged inflows in month i-1, i=2,...i-4, respectively

STG = storage at the beginning of month i

SUM1 = (STG + QFL)

CRP = cross product (STG · QFL).

Results for other target levels are identical for the case of a two-sided quadratic loss function. For the one-sided loss function, the correlations are not significantly different at other target levels except in the months of high flows: February, March and April. In these months, the correlation of release with lagged inflows increases as the target level is raised, while the correlation with current inflow shows a decrease.

Most simple correlations of release with inflows QFL through QFL4 are significant (at the 0.05 level) for both loss functions. In the stepwise regressions, however, neither QFL3 nor QFL4 improve R^2 by more than five percent for the one-sided loss

Table 7.1. Simple Correlation Coefficients of Optimal Monthly Releases with Independent Variables.

MONTH	INDEPENDENT VARIABLES							
	QFL	QFL1	QFL2	QFL3	QFL4	STG	SUMJ	CRP
	Case: Two-Sided Quadratic Loss Function: Target = 75 MGD							
January	0.880	0.715	0.544	0.192	0.190	-0.132	0.835	**0.362**
	(0.0001)	(0.0001)	(0.0001)	(0.0209)	(0.0223)	(0.1152)	(0.0001)	**(0.0001)**
February	0.642	0.752	0.672	0.559	0.196	0.322	0.637	**0.699**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0184)	(0.0001)	(0.0001)	**(0.0001)**
March	0.693	0.573	0.717	0.587	0.506	0.150	0.679	**0.709**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0721)	(0.0001)	**(0.0001)**
April	0.584	0.722	0.530	0.604	0.430	0.284	0.661	**0.789**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0006)	(0.0001)	**(0.0001)**
May	0.835	0.136	0.231	0.134	0.141	-0.161	0.477	**0.406**
	(0.0001)	(0.1044)	(0.4302)	(0.1096)	(0.0912)	(0.054)	(0.0001)	**(0.0001)**
June	0.876	0.830	0.096	0.200	0.091	0.002	0.684	**0.797**
	(0.0001)	(0.0001)	(0.2517)	(0.0162)	(0.2759)	(0.9846)	(0.0001)	**(0.0001)**
July	0.788	0.753	0.762	0.059	0.108	0.226	0.555	**0.823**
	(0.0001)	(0.0001)	(0.0001)	(0.4834)	(0.1965)	(0.0066)	(0.0001)	**(0.0001)**
August	0.567	0.640	0.637	0.619	0.110	0.258	0.472	**0.698**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.1877)	(0.0018)	(0.0001)	**(0.0001)**
September	0.157	0.420	0.435	0.328	0.316	0.202	0.237	**0.235**
	(0.0610)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.015)	(0.0043)	**(0.0045)**
October	0.149	0.144	0.414	0.454	0.360	0.077	0.088	**0.168**
	(0.0749)	(0.0861)	(0.0001)	(0.0001)	(0.0001)	(0.3613)	(0.2928)	**(0.0449)**
November	0.704	0.277	0.169	0.317	0.368	-0.234	0.017	**0.222**
	(0.0001)	(0.0008)	(0.0430)	(0.0001)	(0.0001)	(0.0048)	(0.8442)	**(0.0076)**
December	0.890	0.741	0.253	0.171	0.091	-0.309	0.686	**0.107**
	(0.0001)	(0.0001)	(0.0023)	(0.0401)	(0.2766)	(0.0002)	(0.0001)	**(0.2031)**
	Case: One-Sided Quadratic Loss Function: Target = 75 MGD							
January	0.916	0.743	0.584	0.213	0.149	0.396	0.889	**0.973**
	(0.0001)	(0.0001)	(0.0001)	(0.0106)	(0.0752)	(0.0001)	(0.0001)	**(0.0001)**
February	0.758	0.647	0.590	0.572	0.294	0.601	0.833	**0.947**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0003)	(0.0001)	(0.0001)	**(0.0001)**
March	0.879	0.570	0.408	0.342	0.329	0.506	0.894	**0.978**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	**(0.0001)**
April	0.919	0.578	0.338	0.203	0.158	0.471	0.900	**0.983**
	(0.0001)	(0.0001)	(0.0001)	(0.0146)	(0.0583)	(0.0001)	(0.0001)	**(0.0001)**
May	0.957	0.045	0.049	0.023	0.051	0.119	0.797	**0.971**
	(0.0021)	(0.5934)	(0.5579)	(0.7847)	(0.5442)	(0.1539)	(0.0001)	**(0.0001)**
June	0.961	0.851	-0.022	0.039	-0.075	0.284	0.851	**0.977**
	(0.0001)	(0.0001)	(0.798)	(0.6433)	(0.3716)	(0.0006)	(0.0001)	**(0.0001)**
July	0.908	0.708	0.677	-0.011	-0.077	0.179	0.620	**0.924**
	(0.0001)	(0.0001)	(0.0001)	(0.899)	(0.3612)	(0.0314)	(0.0001)	**(0.0001)**
August	0.889	0.610	0.523	0.363	-0.087	0.125	0.515	**0.914**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.298)	(0.1347)	(0.0001)	**(0.0001)**
September	0.138	0.247	0.202	0.1979	0.227	0.230	0.254	**0.168**
	(0.0998)	(0.0029)	(0.0151)	(0.0174)	(0.0063)	(0.0055)	(0.0021)	**(0.0442)**
October	0.079	0.098	0.078	0.095	-0.006	0.224	0.228	**0.114**
	(0.3462)	(0.2413)	(0.3539)	(0.2586)	(0.9463)	(0.007)	(0.006)	**(0.1724)**
November	0.420	0.058	0.095	0.052	0.029	0.237	0.356	**0.475**
	(0.0001)	(0.4937)	(0.2561)	(0.5382)	(0.7342)	(0.0042)	(0.0001)	**(0.0001)**
December	0.862	0.696	0.149	0.153	0.037	0.347	0.774	**0.944**
	(0.0001)	(0.0001)	(0.0751)	(0.0664)	(0.6576)	(0.0001)	(0.0001)	**(0.0001)**

NOTE: Figures in parentheses give the probability of rejecting the hypothesis that the correlation is zero. For example, at a 5%. level of significance, any probability greater than 0.05 would indicate a zero correlation.

function, and this is also true in most cases for the two-sided loss function.

Storage in the beginning of spill months - February, March and April, shows a higher correlation with release for the one-sided loss function than for the two-sided case. This is expected because large releases are not penalized under a one-sided quadratic loss function. Therefore, spills of greater frequency and magnitude will be observed when a one-sided loss function is substituted for a two-sided loss function. The correlation with storage is attributed to these spills. For the remaining months, storage alone is not a significant variable for either loss function.

Figures 7.2 and 7.3 illustrate, for the two loss functions, the relationship between expected monthly releases and expected monthly storage volumes. Reservoir storages and release in each month are adjusted to meet the target release requirements in the months of low flows. Young (1966), in his study of annual reservoir release policies, found that optimal expected releases were independent of target for the case of a two-sided loss function. The same conclusion is reached in the present study for monthly releases, as shown in Figure 7.2(b).

Regression Models Tested: The general forms of linear and nonlinear policies tested through regression analysis are the following:

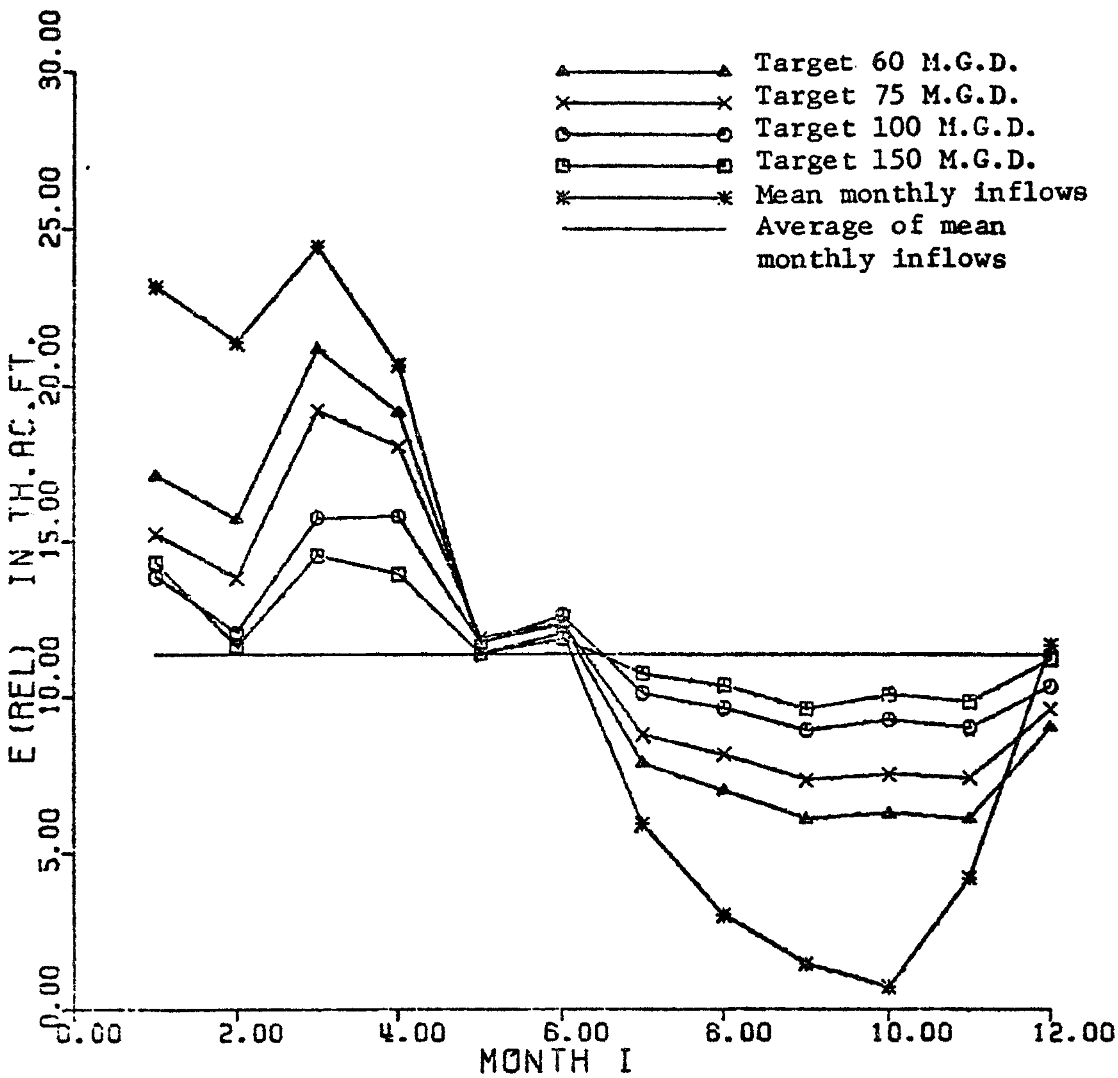


Figure 7.2(a): Expected Monthly Releases Under a One-Sided Quadratic Loss-Function.

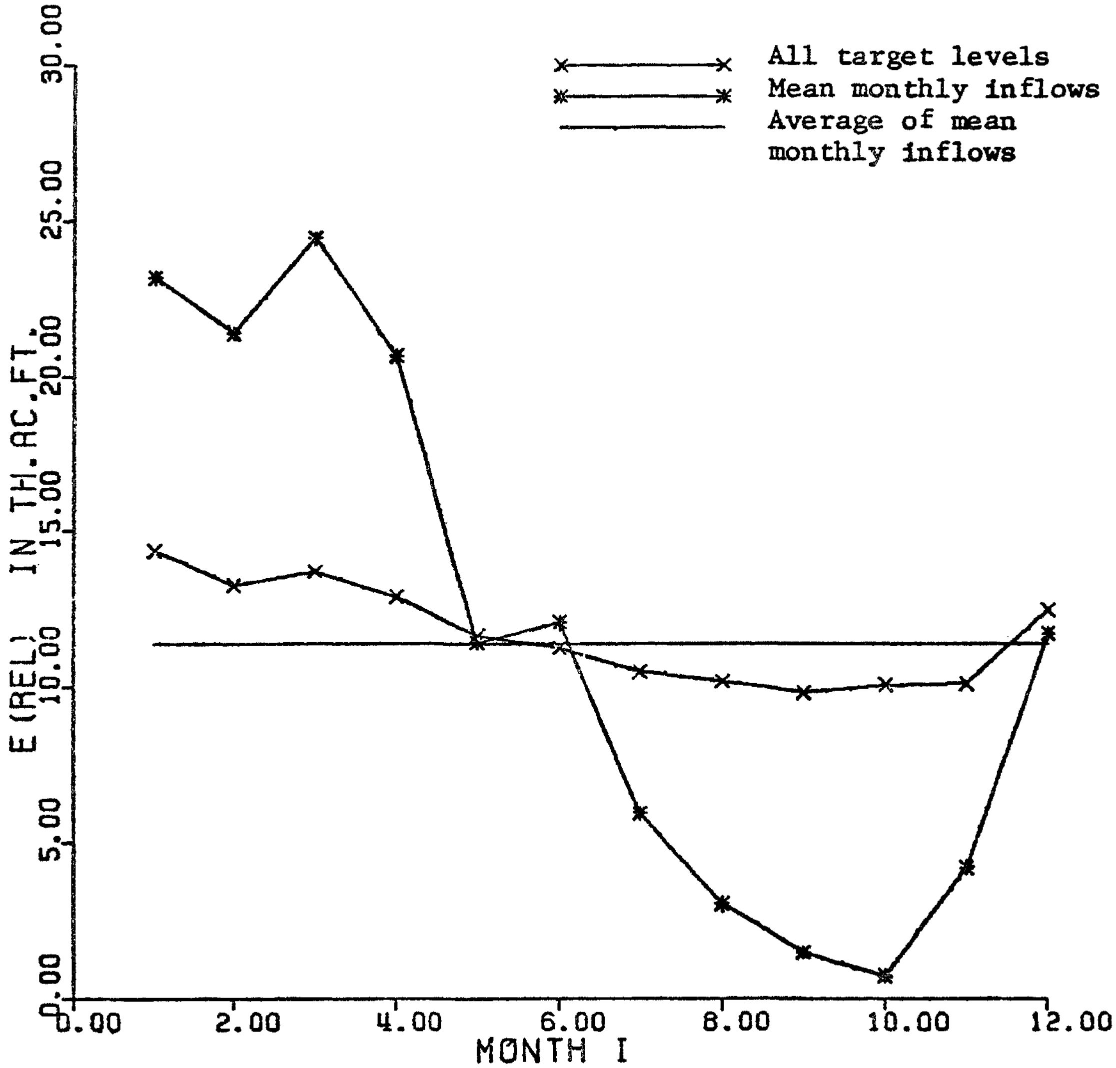


Figure 7.2(b): Expected Monthly Releases Under a Two-Sided Quadratic Loss-Function.

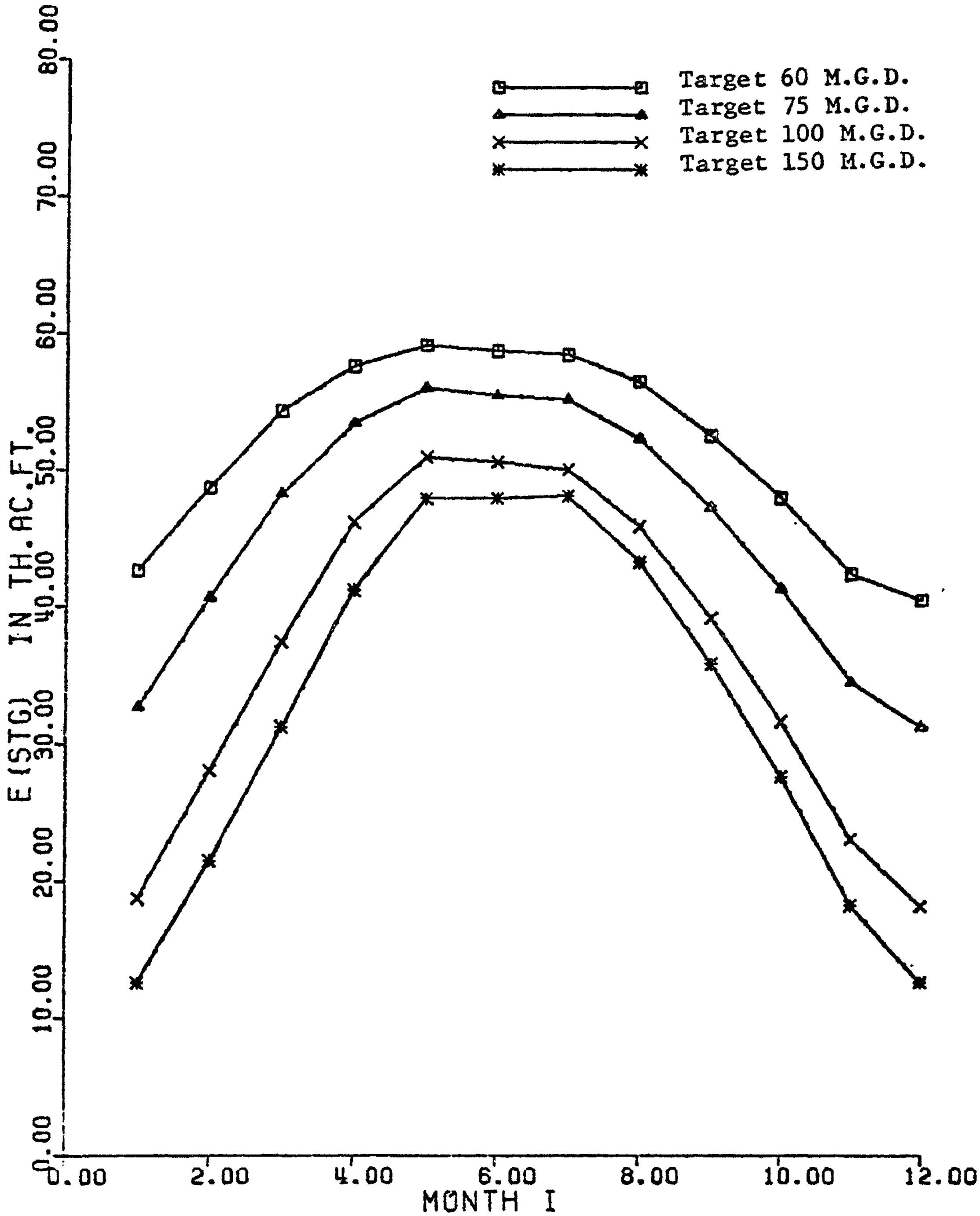


Figure 7.3(a): Expected Monthly Reservoir Storage Under a One-Sided Quadratic Loss-Function.

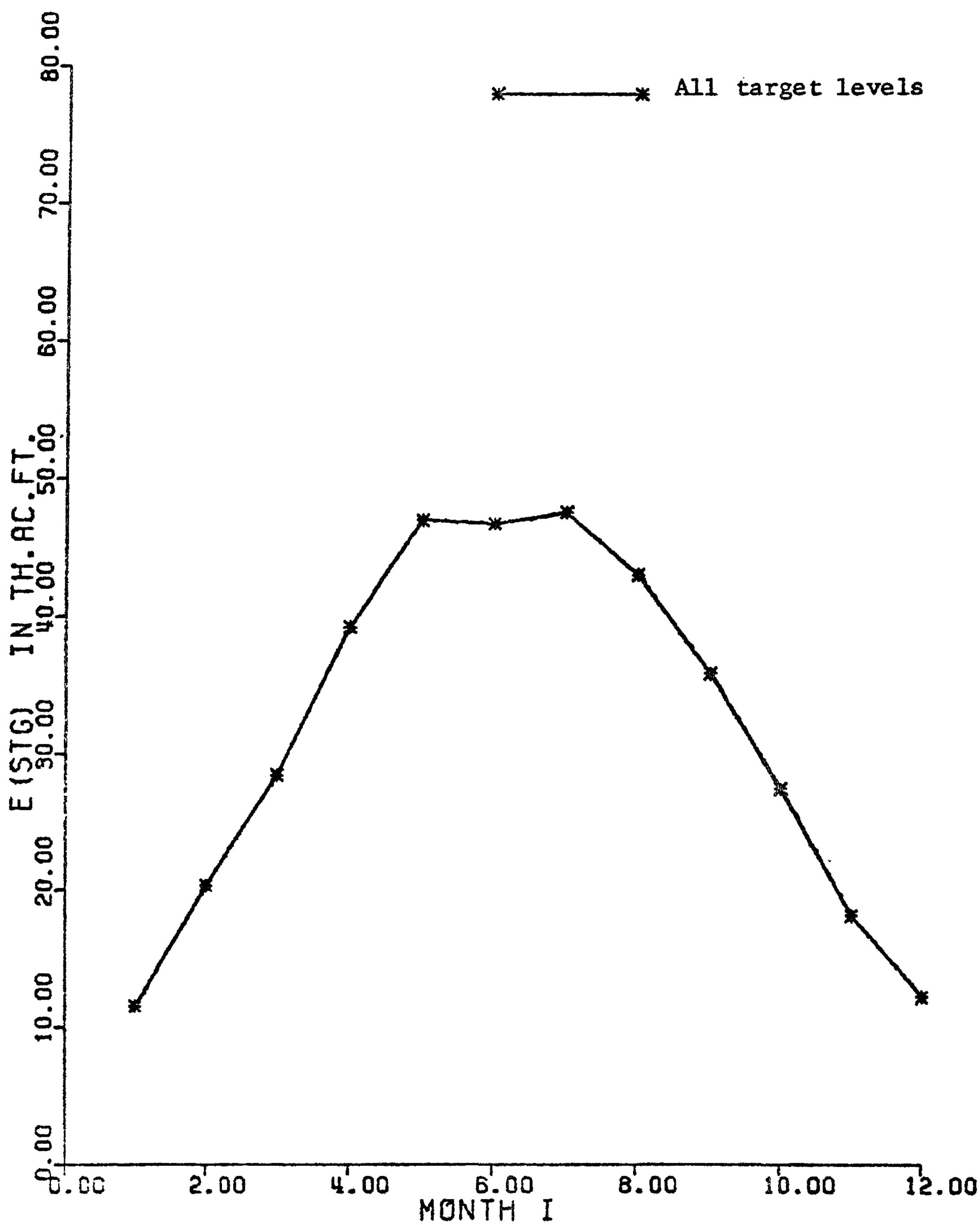


Figure 7.3(b): Expected Monthly Reservoir Storage Under a Two-Sided Quadratic Loss-Function.

Linear Model M1

$$REL = B_0 + B_1(QFL) + B_2(STG) + B_3(QFL1) \ldots + B_6(QFL4).$$

Nonlinear Model M2

$$REL = B_0 + B_1(SUM1) + B_2(SUM2) + B_3(SUM3)$$

Nonlinear Model M3

$$REL = B_0 + B_1(CRP),$$

where REL = Release in month i;

QFL = Inflow in month i;

STG = Storage at the beginning of month i;

QFL1, QFL2...QFL4 = Lagged inflows in month i-1, i=2...i-4, respectively;

SUM1 = (QFL + STG);

SUM2 = $(QFL + STG)^2$;

SUM3 = $(QFL + STG)^3$; and

CRP = $(QFL \cdot STG)$.

All units are in thousand acre-feet.

Regression Results: Appendix H summarizes the linear and nonlinear policies derived by regressing the optimal releases on selected independent variables. Since for the two-sided quadratic loss function the policies are independent of the target level, only the results at a target of 75 MGD are presented.

The tables in Appendix H provide data on two important steps of the step-wise regression procedure. Step 1 includes only the first, and consequently the most significant, variable brought into the model. In this case, R, the square root of the coefficient of determination, will be equal to the simple correlation between the dependent variable and the independent variable admitted at this step. Step 2 corresponds to the step in the regression procedure where addition of another independent variable does not improve R^2 more than 0.05. From now on Step 1 policies will be referred to as "simple" models; while policies at Step 2 are designated as "complete" regression models.

On examining the complete regression models in Tables H1-H4, it may be concluded, based on a maximum R^2 criterion, that

a) Except for the month of April, linear policies, M1, are generally as good as, or better than, nonlinear policies M2 and M3 for the two-sided quadratic loss function.

b) For the one-sided loss function, nonlinear policies are more appropriate than linear policies.

These conclusions are valid at all target levels. It will be demonstrated in the next section, however, that the maximum R^2 criterion does not always provide the best operational model, although the above conclusions are still true.

Prediction equations for policies in the low-flow months of September, October, and November have low R^2 values. This is because a constant release policy is optimal in these months. Consequently, none of the independent variables is very significant.

Simulation Results: In this section, linear and nonlinear monthly policies, and variations of these, are simulated to examine their performance as measured by the average annual loss. Twenty inflow sequences of 150 years each, and reservoir design parameters mentioned in previous chapters, are used in the simulations. Tables 7-2 and 7-3 summarize the results. The standard policy is also included for comparison. Since the direction of results remains unchanged as between the various policies for the two-sided loss function, only results at a 75 MGD (3.3 m^3/s) presented in Table 7-2.

Table 7-2: Performance of Policies Derived Under a Two-Sided Quadratic Loss Function (Target = 75 MGD)

Policy	Average Loss/Year
1. Standard Policy	1786.791
2. Linear Model M1 with QFL only	1155.608
3. Linear Model M1 with QFL1 only	1246.432
4. Complete Linear Model M1	1132.294
5. Nonlinear Model M2 with SUM1 only	1158.782
6. Nonlinear Model M2 with SUM2 only	1145.313
7. Nonlinear Model M2 with SUM3 only	1234.358
8. Complete Nonlinear Model M2	1318.162
9. Simple Nonlinear Model M2	1183.364
10. Nonlinear Model M3	1152.370

Table 7-3: Performance of Policies Derived Under a One-Sided Quadratic Loss Function (units in average loss per year)

Model Type	Target Levels		
	75 MGD	100 MGD	120 MGD
1. Standard Policy	1.024	40.811	156.578
2. Linear Model M1 with QFL only	29.477	77.173	151.978
3. Linear Model M1 with QFL1 only	47.368	94.906	167.759
4. Complete Linear Model ML	24.677	64.196	137.567
5. Nonlinear Model M2 with SUM1 only	148.734	65.062	138.182
6. Nonlinear Model M2 with SUM2 only	11.794	39.959	105.157
7. Nonlinear Model M2 with SUM3 only	7.990	29.992	93.486
8. Complete Nonlinear Model M2	7.466	31.167	94.465
9. Simple Nonlinear Model M2	8.392	28.935	91.993
10. Nonlinear Model M3	12.038	36.610	107.401

The following inferences can be made from the case study for a two-sided quadratic loss function.

1. Linear and nonlinear policies derived using the dynamic programming regression technique are optimal compared to the standard policy at all target levels above or below the mean annual inflow.
2. For the linear model, M1, addition of variables (and consequently a higher R^2) yields better performance

than the simplified models in one significant variable. The reverse is true for the nonlinear model, M2, i.e., addition of variables worsens performance in this case.

3. Although the release policies, both linear and nonlinear, are target independent, there is a particular target level at which the average loss/year is a minimum. This level is close to the mean annual inflow.
4. The complete linear model, M1, is the best prediction model; however, the differences in optimal losses between this model and some of the restricted variable entry (and simple) forms of model M2, as well as nonlinear model M3, are not large.

Since policies under a one-sided quadratic loss function are target dependent, simulation results for target levels of 75 MGD, 100 MGD, and 120 MGD are presented in Table 7-3. The ratio of these target levels to mean annual inflow are 0.599, 0.799, and 0.958, respectively. The mean detention time of the reservoir, defined as the ratio of the reservoir active capacity to the mean annual inflow, is 0.440 years.

Some important conclusions may be drawn for the one-sided quadratic loss function:

1. The standard policy is optimal at a target level of 75 MGD (3.3 Cu.m/sec). This target level is the current draft at Hoover Reservoir and closely represents the safe yield of the reservoir (68 MGD for a 1 in 50 year

shortage). At higher targets, nonlinear policies, M2 and M3, are optimal. For targets lower than 75 MGD, the standard policy remains favorable.

2. Simple and complete nonlinear policies, M2 and M3, are optimal compared to the complete linear policy, M1, at all target levels.
3. As in the case of a two-sided objective function, addition of variables (improvement in R^2) for model M1 improves reservoir performance. Also, simplification of nonlinear policy, M2, by omitting significant variables sometimes slightly improves performance. For example, the simple nonlinear model M2 has a smaller average loss per year than the complete nonlinear model M2 for target levels of 100 and 120 MGD. In general, however, such restriction of variable entry greatly worsens performance for the case of a one-sided objective function. The latter situation is opposite to that found for the case of a two-sided objective function.
4. The simple nonlinear model, M2, is the best policy at high target levels, while the standard policy is best at low target levels (relative to the safe yield estimate).

The improvement in performance of nonlinear policies by accepting regression policies with lower coefficients of determination, R^2, is demonstrated more explicitly in Tables 7-4 and 7-5. Nonlinear model M2 is used as an example for both loss functions.

Table 7-4: Simplification of Model M2 for a Two-Sided Quadratic Loss Function (Target = 100 MGD)

	Procedure 1			Procedure 2		
Step No.	Month Simplified	Average Loss Per Year	Total Percent Loss Reduction	Average Loss Per Year	Percent Loss Reduction	Percent Loss in R^2
1	Complete Model	1138.319	----	1138.319	---	---
2	March	1115.749	1.983	1115.749	1.983	2.90
3	April	1059.396	6.933	1081.551	4.987	11.30
4	May	1047.259	8.000	1128.899	0.828	15.30
5	June	1018.805	10.499	1121.710	1.459	8.00
6	July	1010.355	11.242	1129.059	0.814	13.40
7	August	1003.497	11.844	1132.208	0.537	7.80
				sum =	10.608	

Table 7-5: Simplification of Model M2 for a One-Sided Quadratic Loss Function (Target = 100 MGD)

		Procedure 1		Procedure 2		
Step No.	Month Simplified	Average Loss Per Year	Total Percent Loss Reduction	Average Loss Per Year	Percent Loss Reduction	Percent Loss in R^2
1	Complete Model	31.167	---	31.167	---	---
2	May	30.588	1.954	30.558	1.954	9.60
3	July	29.168	6.414	29.191	6.340	23.70
4	August	28.935	7.161	30.953	0.687	29.40

The months missing in Tables 7-4 and 7-5 have only one significant variable.

When substituting simplified policies in each month, two procedures are adopted. In the first, the simple policy substituted at each step is retained. Thus at the last step, the complete policy model is reduced to a simple policy model for each month. In the second procedure, only one month has a simplified policy at any one time. The loss in R^2 is therefore attributed to the month simplified at each step of the latter procedure.

From Tables 7-4 and 7-5, it may be concluded that significant reduction in the average loss can be achieved using procedure 1 as described above. Use of procedure 2 serves to indicate the degree of interdependence of monthly release policies over the limited range of changes made in going from complete to simple monthly release models. The improvements given under procedure 2 sum to 10.608 percent, whereas procedure 1 gives an 11.844 percent cumulative improvement, for a two-sided loss function. Similar figures for the case of a one-sided loss function are 8.981 percent and 7.161 percent, respectively. Incremental improvements in adding simplified policies under procedure 1 also closely agree with the improvements given by one-at-a-time substitutions under procedure 2. This would serve to indicate a high degree of independence between monthly release policies. Such information is useful when determining optimal monthly policies when strict adherence is not given to the R^2 criterion. It should be emphasized, however, that the degree to which monthly policies are changed in going from complete to simple regression models is not large. More severe policy changes may, and undoubtedly would, result in a higher degree of interdependence.

Effect of Mean Detention Time on Reservoir Performance Under a Two-Sided Loss Function: All the results presented in this chapter so far apply to the existing Hoover Reservoir capacity, which has a mean detention time of 0.44 years. Tables 7-6(a)-(b) summarize the overall performance of linear and nonlinear policies relative to the standard policy for other selected detention times. Based on regression results (not presented here) and the performance characteristics illustrated in Tables 7-6(a)-(b), it can be concluded that:

1. Linear monthly release policies in the current and previous periods' inflows are optimal compared to nonlinear policies.
2. Inflows in previous periods become more important in predicting releases in any given period as the mean detention time increases. This is expected since a higher mean detention time implies a reservoir with a larger capacity, allowing releases in any period to be more dependent on past inflows through increased reservoir regulation.

Table 7.6(a): Comparison of Average Loss/Year At Various Mean Detention Times in Years for a Two-Sided Quadratic Loss Function*

	Mean Detention Time (years)		
Release Policy	1.0	0.44**	0.20
Standard Policy	1778.881	1786.791	1937.966
Complete Linear Model, M1	715.229	1132.294	1655.413
Complete Nonlinear Model, M2	964.647	1318.162	1745.566
Nonlinear Model M3	750.245	1152.370	1701.691

*Mean annual inflows, μ = 136.514 thousand acre-feet
Target = 75 MGD

**Existing Hoover Reservoir capacity

Table 7.6(b): Comparison of Percentage Reduction in Average Loss/Year Relative to Standard Policy at Various Mean Detention Times in Years for a Two-Sided Quadratic Loss Function

	Mean Detention Time (years)		
Release Policy	1.0	0.44	0.20
Standard Policy	---	---	---
complete Linear Model, M1	59.79	36.63	14.58
Complete Nonlinear Model, M2	45.77	26.23	9.93
Nonlinear Model, M3	57.82	35.51	12.19

3. The percentage improvement in the performance of the dynamic programming policies over the standard policy increases with an increase in the mean detention time. This is undoubtedly due to the nature of these policies, since the standard policy incorporates only the current inflow and storage in making release decisions at all levels of the mean detention time. In contrast, the dynamic programming policies allow the incorporation of more of the history of the system, by reflecting the increased dependence of current releases on past inflows at higher mean detention times.

Annual Model: The dynamic program developed in Chapter 5 is solved assuming that each stage, i, represents a year in the operation of the reservoir. The objective is to compare annual release policies pertaining to the case example presented in this study with those derived by Young (1966). In both cases the relative magnitude of important reservoir design parameters is maintained at the same level, as shown in Table 7-7.

The optimal linear and nonlinear release policies, obtained using a two-sided quadratic loss function, are illustrated in Table 7-8. In both models optimal releases are independent of the target level. For the model linear in inflow and storage, the regression coefficients of Young's policy are within the 99 percent confidence limits of the coefficients derived in the present

Table 7-7: Comparison of Design Parameters for Annual Reservoir Operation

Parameter	Young's Study*	Present Study
1. Loss Function	$\sum_t (X_t - T)^2$	$\sum_t (X_t - T)^2$
2. Mean Inflow, μ	10.0	135.721 th.ac.ft.
3. Reservoir Capacity, c	10.0	135.8965 th.ac.ft.
4. Target, T	7.0	95.1276 th.ac.ft.
5. Ratio, T/μ	0.70	0.70
6. Coefficient of Variation, C_v	0.30	0.44

*Units on μ, c and T are arbitrarily selected.

Table 7-8: Comparison of Reservoir Release Policies for Annual Reservoir Operation

Young's Study				Present Study			
B_o	B_1	B_2	R^2	B_o	B_1	B_2	R^2
1. Linear Policy: REL = $B_o + B_1(STG) + B_2(QFL)$							
6.848	0.065	0.290	0.41	83.101	0.106	0.325	0.47
2. Nonlinear Policy: REL = $b_o + B_1(SUM1) + B_2(SUM2)$							
9.926	-0.159	0.0104	0.23	81.524	0.257	---	0.33

study. Similar comparison for the nonlinear model could not be made since the stepwise regression results showed the second order term to be insignificant. However, the coefficient of determination, R^2, for both the linear and nonlinear models is in close agreement with those derived by Young.

In his study of annual release policies, Young concluded that, for a smooth, convex loss function, setting $X_t = \mu$ (whenever possible) is a near optimal policy, where μ is the mean annual inflow and X_t is the release in period t. This is also shown to be true for annual release policies developed in this study, since for both the linear and nonlinear policies presented in Table 7-8 the expected value of releases is close to the mean annual inflow. Figure 7-4 shows optimal releases for the annual model compared to the standard policy.

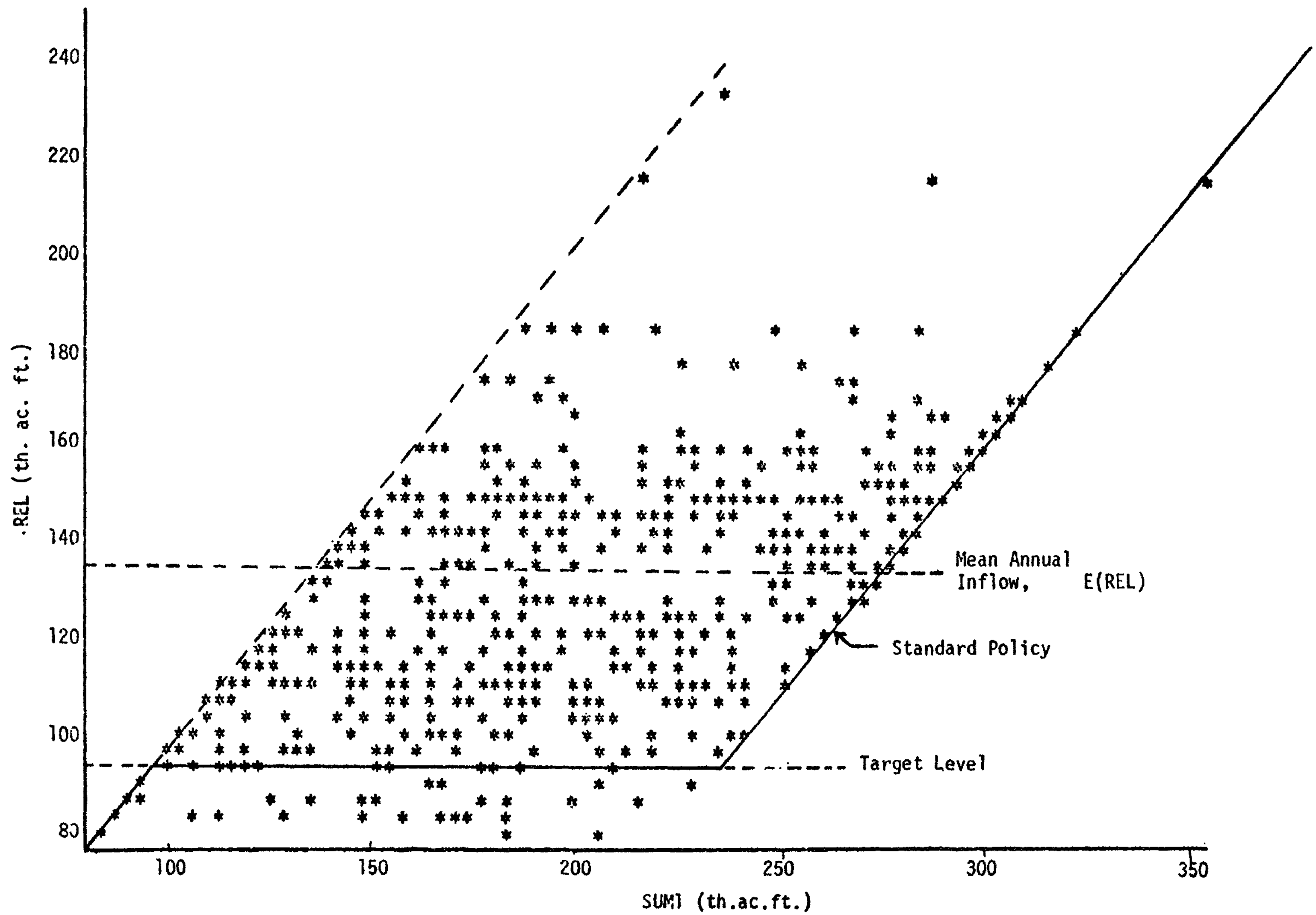


Figure 7-4: Plot of Optimal Annual Releases (SUM = Inflow + Storage)

CHAPTER VIII

COMPARISON OF OPTIMIZATION MODELS

In the previous two Chapters, optimal monthly release policies were derived for a single multi-purpose reservoir, using chance constrained linear programming and dynamic programming, respectively. A comparison of policies derived under these two approaches is the major emphasis of this Chapter. Linear release policies obtained from the dynamic programming regression methodology are selected to correspond with the general form of the linear decision rule, defined in Chapter 5 (Equation 5-7). Simulation results, summarizing important statistics of various performance measures are presented. In the final section, the trends in the slope coefficient of bivariate-regression forms are examined over a range of mean detention times. It is hoped that such trends would be useful in predicting release policies at a general site without requiring a solution of the dynamic programming algorithm.

Comparison of Optimal Release Policies: Tables 8.1 and 8.2 summarize special forms of optimal monthly release policies derived under the chance-constrained linear programming and dynamic programming-regression methodologies (corresponding to mean detention time of 0.44 and a target of 75 MGD). These policies can be mathematically defined as follows:

Table 8.1: Special Forms of Optimal Release Policies[a]

Month	Model LDR1 $X_i = R_{i-1} + b_{i-1} - b_i$		Model LDR2 $X_i = R_i + b_{i-1} - b_i$		Model DP1 $X_i = B_o + B_1 (R_{i-1})$			Model DP2 $X_i = B_o + B_1 (R_i)$		
	b_i	$b_{i-1} - b_i$	b_i	$b_{i-1} - b_i$	B_o	B_1	R[b]	B_o	B_1	R[c]
January	-0.5786	2.5772	5.7351	-3.4980	5.1512	0.8395	0.715	2.3633	0.5178	0.880
February	2.9179	-3.4965	14.7825	-9.0474	7.5136	0.2479	0.752	2.7241	0.4924	0.643
March	11.9668	-9.0489	28.2736	-13.4911	5.3884	0.3811	0.573	4.8304	0.3628	0.693
April	25.4579	-13.4911	36.3613	-8.0877	5.3641	0.3076	0.722	6.3952	0.3139	0.584
May	33.5456	-8.0877	35.9176	0.4437	10.3197	0.0634	0.188	6.3433	0.4634	0.836
June	33.1019	0.4437	35.1269	0.7907	6.1919	0.4431	0.830	5.9305	0.4403	0.876
July	32.3112	0.7907	30.7060	4.4209	7.2251	0.2706	0.753	7.4973	0.5048	0.787
August	27.8903	4.4209	24.0677	6.6383	8.5083	0.2830	0.640	9.3153	0.2885	0.568
September	21.2520	6.6383	17.0828	6.9849	9.3160	0.1745	0.421	9.6282	0.1460	0.158
October	14.2671	6.9849	10.2619	6.8209	9.8698	0.1340	0.145	9.7537	0.4259	0.148
November	7.4462	6.8209	4.8143	5.4476	9.4184	0.9547	0.278	8.0988	0.4819	0.704
December	1.9986	5.4476	2.2371	2.5772	4.6488	1.8615	0.741	5.4972	0.5951	0.890

a. Existing Hoover Reservoir design is assumed, (C = 86.1228 th. ac. ft.; mean detention time = 0.44) all units in thousand acre-feet, target = 75 MGD.

b. Correlation of release with the current inflow, ρ_1

c. Correlation of release with previous period's inflow, ρ_2

Table 8.2: General Linear Release Policy**

Model DP3: $X_t = B_o + B_1 (R_t) + B_2 (R_{t-1})$

Month	B_o	B_1	ρ_1^*	B_2	ρ_2^*	R	ρ_3^*
January	1.3433	0.4224	0.880	0.2945	0.715	0.900	0.647
February	1.8665	0.3204	0.643	0.1958	0.752	0.846	0.378
March	2.0844	0.2850	0.693	0.2171	0.573	0.749	0.464
April	3.5834	0.1611	0.584	0.2443	0.722	0.768	0.496
May	5.0339	0.4634	0.835	0.0634	0.136	0.846	0.000
June	5.6228	0.3057	0.876	0.1694	0.830	0.892	0.843
July	6.9323	0.3271	0.788	0.1341	0.753	0.826	0.744
August	8.5642	0.1270	0.567	0.2089	0.640	0.666	0.672
September	9.1489	0.1207	0.157	0.1709	0.420	0.439	0.066
October	9.6516	0.3495	0.149	0.1076	0.144	0.187	0.232
November	8.1385	0.4909	0.704	-0.1050	0.277	0.704	0.429
December	6.0583	0.6832	0.890	-0.3793	0.741	0.893	0.873

** Existing Hoover Reservoir design is assumed (C = 86.1228 th. ac. ft), all units in thousand acre-feet, target - 75 MGD

* ρ_1 and ρ_2 are correlation coefficents of monthly release, X_t with the current and previous months inflows, R_t and R_{t-1}, respectively. ρ_3 the correlation between the inflows R_t and R_{t-1}. These estimates are based on a sample size of 148.

Model LDR1 $X_t = s_{t-1} - b_i$ (original)

$X_t = (b_{i-1} - b_i) + R_{t-1}$ (transformed)

or, $X_t = B_o + R_{t-1}$ (8.1)

Model LDR2 $X_t = s_{t-1} + R_t - b_i$ (original)

$X_t = (b_{i-1} - b_i) + R_t$ (transformed)

or, $X_t = B_o + R_t$ (8.2)

Model DP1

$$X_t = B_o + B_1 (R_{t-1}) \quad (8.3)$$

Model DP2

$$X_t = B_o + B_1 (R_t) \quad (8.4)$$

Model DP3

$$X_t = B_o + B_1 (R_t) + B_2 (R_{t-1}) \quad (8.5)$$

where Bo, B_1, and B_2 are regression coefficients.

The forms of the release policies represented by Models DP1, DP2, and DP3 are restricted to resemble the general forms of the linear decision rule. This is done to examine the performance of the linear decision rule under a wider range of possible values for the parameter, λ_i. Since the chance constrained policies implicitly assume a two-sided loss function (see Appendix E, Chapter 6) the dynamic programming policies presented in this Chapter are derived using a similar loss function. Policy DP1 corresponds to the linear decision rule, LDR1, since both these policies incorporate the inflow in the previous period, R_{t-1}. On the other hand, releases under policies DP2 and LDR2 are based on the current inflow, R_t.

The results presented in Table 8.1 show that:

a) unlike the chance-constrained policies, LDR1 and LDR2, the coefficients associated with the current or previous period's inflow in the policies DP1 and DP2, are not equal to unity, and in general are widely different from unity; and

b) the intercept terms, B_0, in models DP1 and DP2 do not correspond at all to the related terms, $b_{i-1} - b_i$, in models LDR1 and LDR2.

The restricted linear forms of the dynamic programming-regression procedure do not, therefore, correspond in value to those linear forms derived using the chance-constrained approach. This conclusion holds at all target levels, since both the chance-constrained and dynamic programming results are target independent. The same conclusion is true at other reservoir capacitites as well.

Simulation Results: The operation of Hoover Reservoir is simulated using the linear release policies shown in Tables 8.1 and 8.2. A synthetically generated inflow sequence of 148-year duration constitutes inflows into the reservoir. Overall performance measures, based on 20 such simulations, are evaluated to compare operational differences inherent in the release policies.

Tables 8.3 and 8.4 summarize the average monthly releases and storages observed under linear release policies expressed in terms of the current and/or lagged inflows. The releases are adjusted in order that the storage remains within the prescribed limits of SMAX and SMIN. The monthly trends in the average release and storage for policies LDR1, LDR2, DP1 and DP2 are graphically illustrated in

Table 8.3: Average Monthly Releases*

Release Policy

Month	LDR1 (transformed)	DP1	LDR2 (transformed)	DP2	DP3	Complete Linear Model M1
January	15.464	15.644	18.452	14.463	14.698	14.405
February	17.581	13.538	11.916	13.603	13.079	13.085
March	12.386	15.588	11.605	15.588	14.971	14.868
April	12.240	14.969	11.631	14.777	14.527	14.255
May	11.892	12.435	11.732	12.042	11.887	11.928
June	12.317	12.563	12.752	12.236	12.241	12.155
July	12.629	10.300	10.225	10.376	10.515	10.549
August	9.950	9.221	9.515	9.536	9.882	9.994
September	8.995	7.765	8.229	8.178	8.601	8.766
October	7.482	6.573	7.457	7.045	7.410	7.601
November	6.631	7.003	9.313	7.460	7.568	7.700
December	9.198	11.156	13.867	11.452	11.373	11.451

*Actual releases after adjusting storage levels to be within the prescribed storage limits.
All units in thousand acre-feet
Existing Hoover Reservoir design is assumed (C = 86.1228 th. ac. ft.) Target = 75 MGD

Table 8.4: Average of Beginning Monthly Storage*

Month	LDR1 (transformed)	DP1	LDR2 (transformed)	DP2	DP3	Complete Linear Model M1
January	9.043	9.982	2.237	10.859	10.342	9.758
February	15.483	16.242	5.689	18.300	17.548	17.257
March	18.707	23.509	14.578	25.502	25.273	24.977
April	30.724	32.324	27.376	34.317	34.706	34.512
May	38.072	36.942	35.333	39.128	39.766	39.844
June	37.469	35.795	34.889	38.374	39.167	39.205
July	37.113	35.194	34.098	38.099	38.887	39.011
August	30.289	30.698	29.677	33.527	34.177	34.267
September	23.216	24.355	23.039	26.869	27.172	27.105
October	15.468	17.837	16.057	19.938	19.817	19.631
November	8.664	11.942	9.279	13.572	13.085	12.709
December	6.286	9.192	4.220	10.366	9.771	9.264

*Based on actual storages between the prescribed storage limits.
All units in thousand acre-feet
Existing Hoover Reservoir design is used (C = 86.1228 th. ac. ft.)
Target = 75 MGD

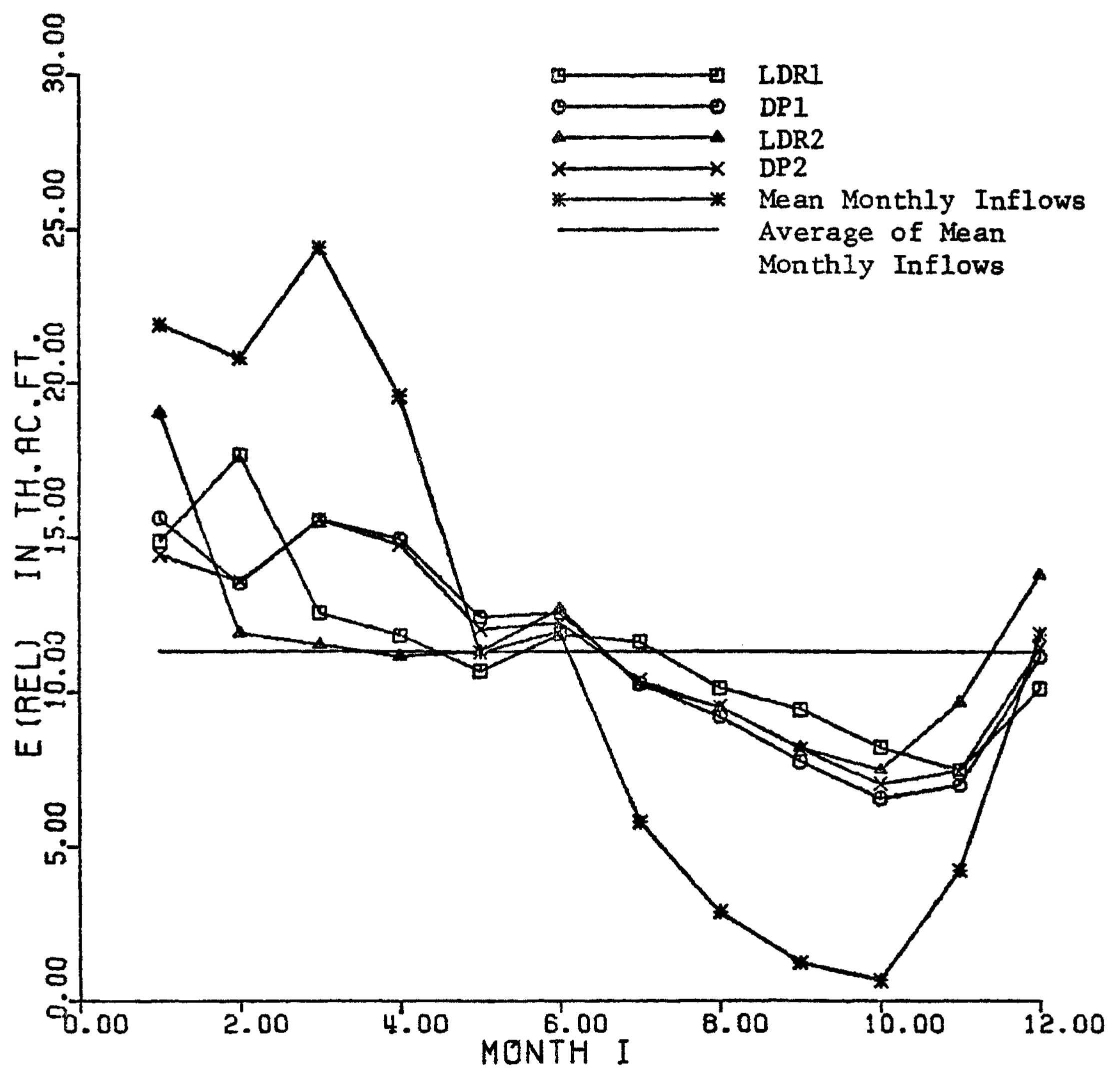


Figure 8.1: Comparison of Monthly Expected Release Under the Linear Decision Rules and the Corresponding Dynamic Programming Policies.

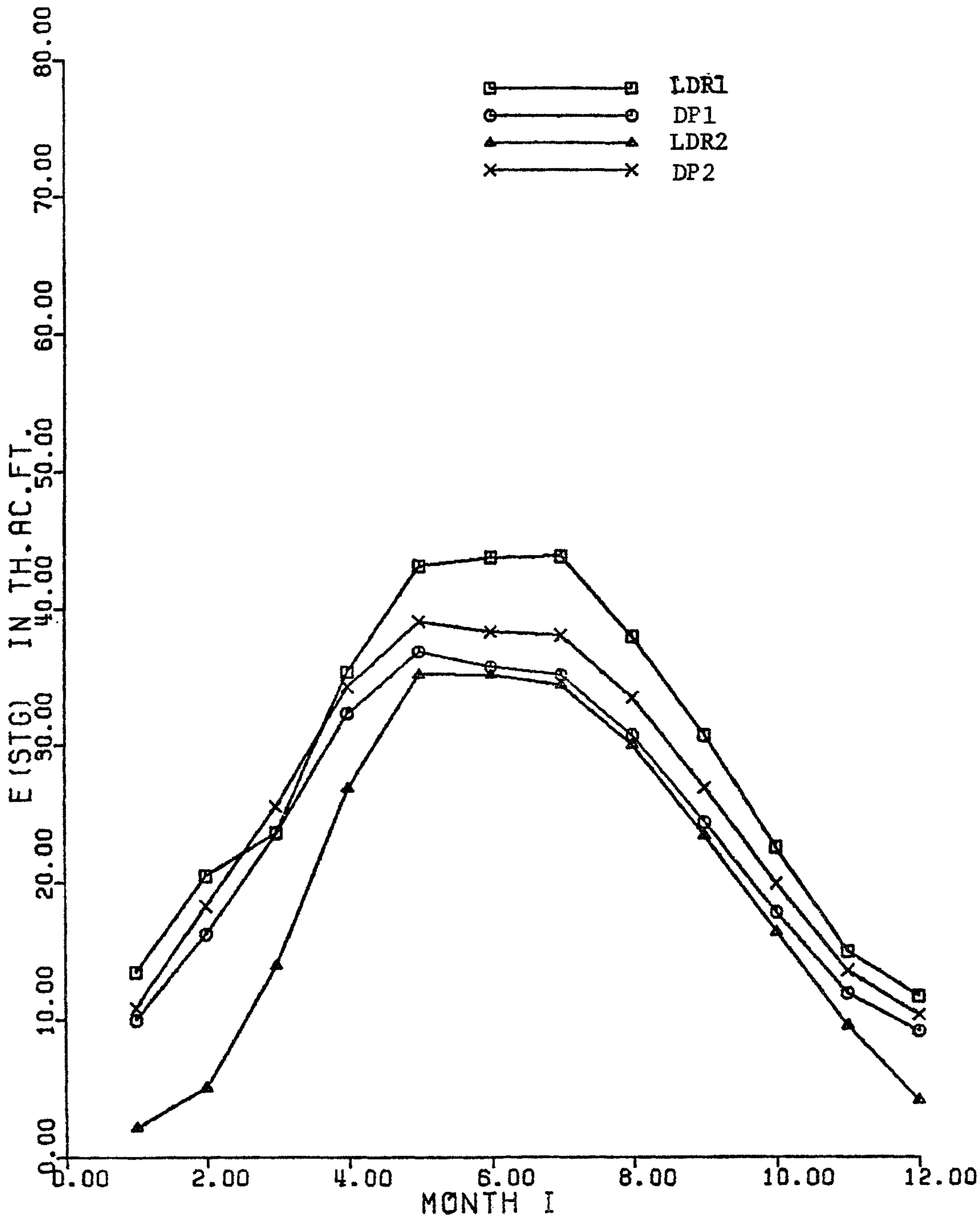


Figure 8.2: Comparison of Monthly Expected Storage Under the Linear Decision Rules and the Corresponding Dynamic Programming Policies.

Figures 8.1 and 8.2, respectively. These results indicate that the pattern of releases and storages associated with the chance-constrained programming policies, LDR1 and LDR2, are quite different from those under the dynamic programming policies, DP1 and DP2. However, it is interesting to observe that a comparison between the dynamic programming policies, DP1, DP2, DP3, and the complete linear model M1, shows the trends in average releases and storages to be in close agreement for all four variations of the dynamic programming results.

Statistics of the average number of shortages, presented in Table 8.5, reflect the reliability with which the target release can be met in each month. It may be recalled from Chapter 6, that the chance-constrained linear programming policies, LDR1 and LDR2, were solved assuming a 50% reliability of satisfying the target release (75 MGD). Results in Table 8.5 indicate that, at a similar target-level of 75MGD, the dynamic programming policies also exhibit percentage monthly shortages which are within the 50% reliability level.

At higher reliability levels, α' (consequently lower minimum guaranteed flows) the optimal reservoir capacities under the chance-constrained models, LDR1 and LDR2, are below the existing Hoover Reservoir capacity of 86.1228 th. ac. ft. Table 8.7 gives the optimal reservoir designs for reliability levels higher than 0.50, corresponding to minimum guaranteed flows less than 75 MGD. Also included are estimates, using Figure 4.2, of safe-yields associated

Table 8.5: Average Monthly Shortages* (Percent)

Month	LDR1 (transformed)	DP1	LDR2 (transformed)	DP2	DP3	Complete Linear Model M1
January	30.78	17.43	35.27	29.87	28.99	29.87
February	33.14	1.05	38.58	4.53	10.54	10.54
March	41.12	0.78	48.78	1.59	6.45	7.80
April	47.50	1.00	41.96	0.17	2.33	6.59
May	42.37	0.78	35.88	1.79	4.29	6.93
June	34.60	4.05	34.87	6.72	6.32	6.08
July	37.13	7.70	30.91	3.85	1.69	1.49
August	32.80	15.54	21.01	10.37	6.12	5.34
September	15.51	25.68	0.10	20.44	15.71	14.39
October	23.55	39.66	29.53	35.10	31.11	30.61
November	25.74	43.01	24.70	39.39	37.94	37.26
December	34.73	38.89	31.35	29.93	28.85	29.29

*Based on actual releases short of the target (75MGD)
Existing Hoover Reservoir design assumed (C = 86.1228 th. ac. ft.)

Table 8.6: Average Monthly Losses per Year*

Month	LDR1 (transformed)	DP1	LDR2 (transformed)	DP2	DP3	Complete Linear Model M1
January	466.302	416.756	698.175	357.789	359.165	353.799
February	487.951	113.750	158.625	107.930	106.813	106.684
March	184.471	180.483	200.283	174.779	163.925	168.464
April	235.335	156.402	157.155	146.065	146.537	146.578
May	157.116	49.129	106.112	58.829	55.651	56.005
June	160.371	122.840	176.776	115.103	113.098	106.786
July	158.785	31.948	44.416	26.938	28.280	27.362
August	40.776	21.105	29.683	17.416	17.742	18.248
September	29.160	14.943	5.769	12.409	12.043	11.949
October	7.974	19.297	0.659	17.330	15.741	16.948
November	3.742	13.290	24.241	17.219	17.119	17.931
December	55.177	106.502	233.518	103.800	102.557	101.837
Total	1987.160	1246.445	1835.412	1155.607	1138.671	1132.591

*under a two-sided quadratic loss function
Existing Hoover Reservoir design is used (C = 86.1228 th. ac. ft.)
Target = 75 MGD

Table 8.7: Optimal Reservoir Design and Reliabilities at Selected Levels of Mean Detention Times.[1]

Mean Detention Time (Years)	Reservoir Capacity, C (th. ac. ft)[2]	Safe Yield (MGD)[3]	Minimum Guaranteed Flow, q_i (MGD)[4]	Reliability, α' of meeting the minimum guaranteed flow, min	Reliability, of meeting the Safe-Yield[5]
Model LDR1					
0.44	86.1228	76.0	75.0	0.50	0.50
0.37	76.4998	70.0	45.0	0.70	0.53
0.35	73.9998	68.0	33.0	0.80	0.55
0.31	68.5998	64.0	23.0	0.90	0.57
0.30	67.6028	62.0	12.0	0.98	0.58
Model LDR2					
0.20	53.0838	52.0	52.0	0.65	-
0.18	49.9998	48.0	45.0	0.70	-
0.14	45.1998	40.0	33.0	0.80	-
0.11	40.1998	33.0	23.0	0.90	-
0.07	34.6968	23.0	12.0	0.98	-

1. Reliability on the flood-control, minimum storage and maximum release chance-constraints, $\alpha^* = 0.83$
2. The maximum w/s usable storage SMAX is obtained from the capacity, C by subtracting the flood-control storage, FMAX = 25.7808 th. ac. ft.
3. Includes evaporation loss.
4. Corresponding to reservoir capacity, C, optimal for LDR models
5. At these levels of reliability, the reliability α^* on the remaining chance-constraints must be adjusted

with the optimal reservoir capacities. For model LDR1, the reliability α^* is adjusted in order that the reservoir capacity, C remains the same for the safe-yield and the minimum guaranteed flow, q_i. However, for model LDR2 such an adjustment is not possible since the capacity is dependent only on q_i. (See Figure 6.2(b)). Consequently, model LDR2 is not analyzed for safe-yield drafts. From the results presented in Table 8.7 it is observed that:

a) With an increase in the reliability, α', the minimum guaranteed flow, as obtained under the chance-constrained models, LDR1 and LDR2 (Equation 5.21), is less than the safe-yield at the corresponding optimal reservoir capacity levels. Consequently, the reliability with which the safe-yield draft can be met is lower than the reliability α' (See Figure 5.2)

b) From the discussion in (a) it can be inferred that the range of mean detention times, for comparing the dynamic programming policies with the chance-constrained programming policies, LDR1 and LDR2, is restricted by the reliability α', since further reduction beyond the 0.50 level*, is unreasonable from a physical point of view. Such lower values are therefore not included in the present comparison.

* Alternatively, draft rates above the 75 MGD level (Figure 5.2)

c) For model LDR2 the mean detention times, particularly for high values of reliability α', are small and therefore unrealistic, but are included for purposes of illustration.

The dynamic programming policies DP1, DP2 and DP3 (Equations 8.3-8.5) are derived using mean detention times underlined in Table 8.7. Based on simulation results, the following conclusions are made.

1) Simulation under the minimum guaranteed flow, q_i:

a) While the chance-constrained programming polices, LDR1 and LDR2 (original forms), yield a probability of shortage in any month within the assumed level of reliability, α', the dynamic programming policies do not meet the target with the desired level of reliability as the mean detention time decreases. However, at a mean detention time of 0.44 years, both the dynamic programming and the chance-constrained programming policies yield reliabilities within the reliabilities level α' (as discussed earlier)

b) For the dynamic programming policies, results suggest that the performance in terms of reliability, α', deteriorates with decreasing mean detention times in the low flow months, while the reverse is true in high low months (January-April).

2) Simulation under the safe-yield:

   a) Although there is a significant reduction in the reliability, α', with which the chance-constrained policy, LDR1, can meet the safe-yield drafts, simulation results indicate that the policies derived under LDR1 do, in fact, satisfy the corresponding reliability levels. Dynamic programming policies fail to do so at the safe yield draft level.

In summary, it appears that, for mean detention times less than 0.44 years, the chance-constrained programming policies do satisfy the imposed reliabilities, α', of meeting the minimum guaranteed flow, q_i and the safe-yield (only model LDR1 is considered for the latter draft level). On the contrary, the dynamic programming policies, under similar reservoir capacity and draft levels, do not meet the reliability α'.

In the next section, trends in the reliability of meeting safe yield drafts under operating policies derived from dynamic programming are investigated in some detail for a wider range of reservoir detention times.

Trends in Safe-Yield Reliability under Dynamic-Programming Policies:

Table 8.8 illustrates the probability of having a monthly shortage under the complete linear model, M1, when the reservoir is operated at selected mean detention times and corresponding safe-yields. As mentioned earlier, since the performance of dynamic programming policies

Table 8.8: Distribution of Average Monthly Shortages (percent) at Various Mean Detention Times and Corresponding Safe-Yields.

Mean Detention Time (Years)	0.20	0.44	1.0
Safe-Yield (MGD)*	52.0	76.0	110.0
Reservoir Capacity (th. ac. ft.)	53.084	86.123	162.295
Month			
January	15.95	29.87	45.14
February	0.64	10.54	28.18
March	0.37	7.80	34.32
April	0.68	6.59	29.05
May	4.46	6.93	9.19
June	16.49	6.08	17.77
July	12.47	1.49	19.26
August	3.41	5.34	20.74
September	13.65	14.39	17.91
October	33.99	30.61	26.35
November	43.72	37.26	38.82
December	26.86	29.29	47.60

* Includes evaporation

DP1, DP2 and DP3 closely agree with the performance under complete linear model M1, results for only the latter case are presented.

From Table 8.9 it can be inferred that, in general, the reliability of meeting the safe-yield deteriorates with increasing mean detention times.

Discussion of Linear Decision Rule Coefficients: For a simple linear regression model , $y = B_o + B_1(x)$, the regression coefficient, B_1, is defined as

$$B_1 = \rho_{x,y} \cdot (\sigma_y/\sigma_x) \tag{8.6}$$

where,

$\rho_{x,y}$ = correlation of the dependent variable y with the independent variable x; and

σ_x, σ_y = standard deviations of x and y, respectively.

Also, the simple correlation coefficient, $\rho_{x,y}$ will be equal to the square root of the coefficient of determination R^2. Thus the coefficient, b_1, in the release policies DP1 and DP2 depends on the correlation coefficient and the ratio of variances of monthly releases with the current or previous period's inflow, respectively. For the chance-constrained linear programming policies LDR1 and LDR2, the assumption of a unit regression coefficient (B_1 = 1 in Equation 8.6) suggests that the variance of release, σ_y, is greater than, or at least equal to, the variance of inflow, σ_x. This must be the case, since, for positive values of B_1, the correlation of release with inflow, ρxy in Equation 8.6 is defined over the range [0, 1] , and B_1

can be equal to 1 only under either of the following two conditions:

a) when the correlation, ρ_{xy}, and ratio of the variances (σ_y/σ_x) are both equal to unity, in which case, $\sigma_y = \sigma_x$; or

b) when the correlation ρ_{xy} is less than unity and (σ_y/σ_x) is greater than unity, in which case $\sigma_y > \sigma_x$.

Table 8.1 shows that for the dynamic programming policies, DP1 and DP2, the regression coefficient, B_1, and the correlation coefficient, ρ_{xy}, are less than unity in most of the months. Also in such cases, the regression coefficient B_1 is less than the correlation coefficient, ρ_{xy}. The above two conditions imply that the ratio of variances σ_y/σ_x in Equation 8.6 is less than unity. Consequently, for these policies the variance of release is less than that of the inflows. This is desirable, since the primary reason for reservoir regulation of streamflows is to reduce the effect of streamflow variability on the releases.

For release policy DP3, (Equation 8.5), the regression coefficients B_1 and B_2 do not sum to unity as in Equation 5.7 (Chapter 5). The degree of dependence between these two coefficients is a function of the correlation of the inflows R_t and R_{t-1}.

<u>Prediction of the Regression Coefficient, B_1:</u> In the last section it was stated that the regression coefficient, B_1, in release policies DP1 and DP2, is a product of the following two parameters:

a) the correlation of releases with the current (model DP1) or previous period's inflow (model DP2); and

b) the ratio of the standard deviation of release to the standard deviation of the current or previous period's inflow.

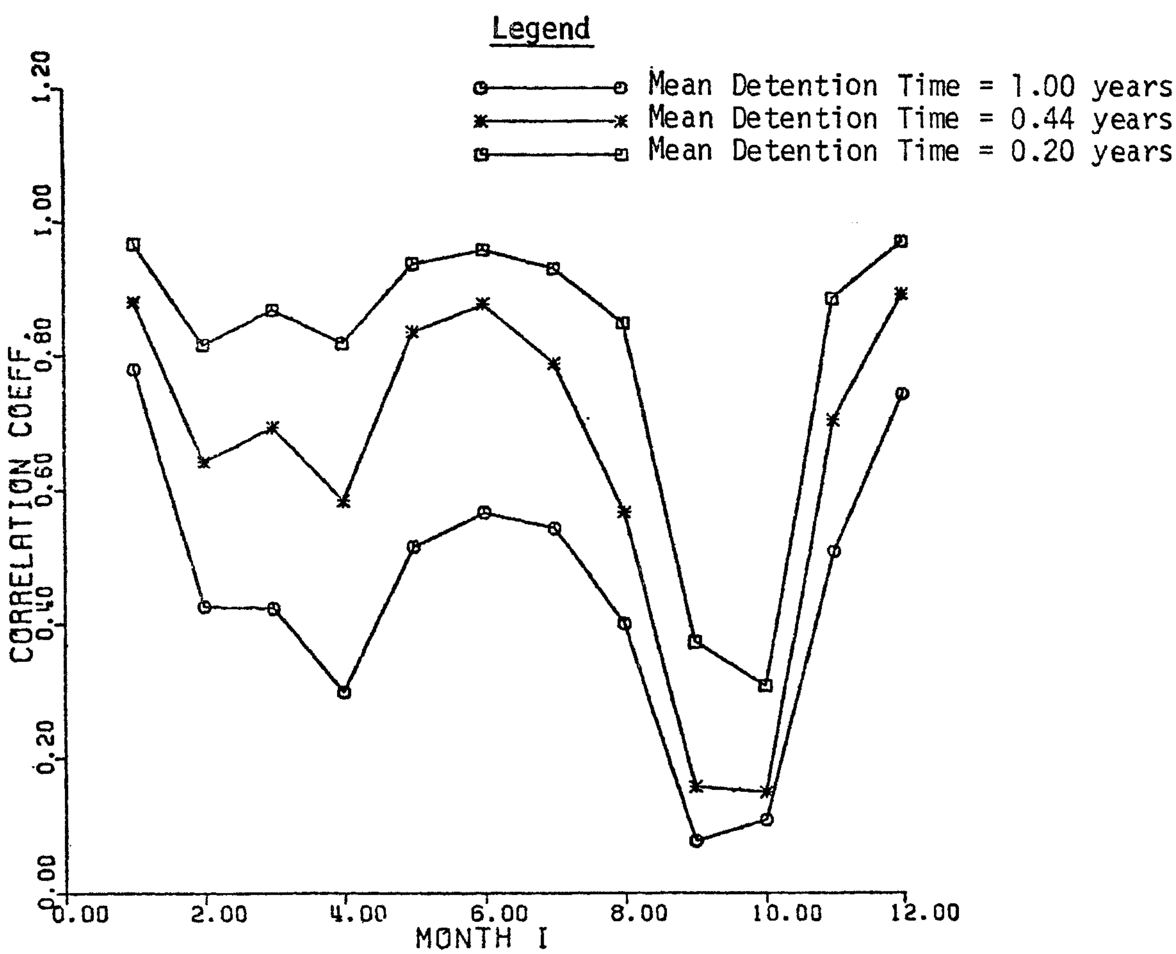


Figure 8.3 (a): Monthly Trends in the Correlation Coefficient of Optimal Release and Current Inflow.

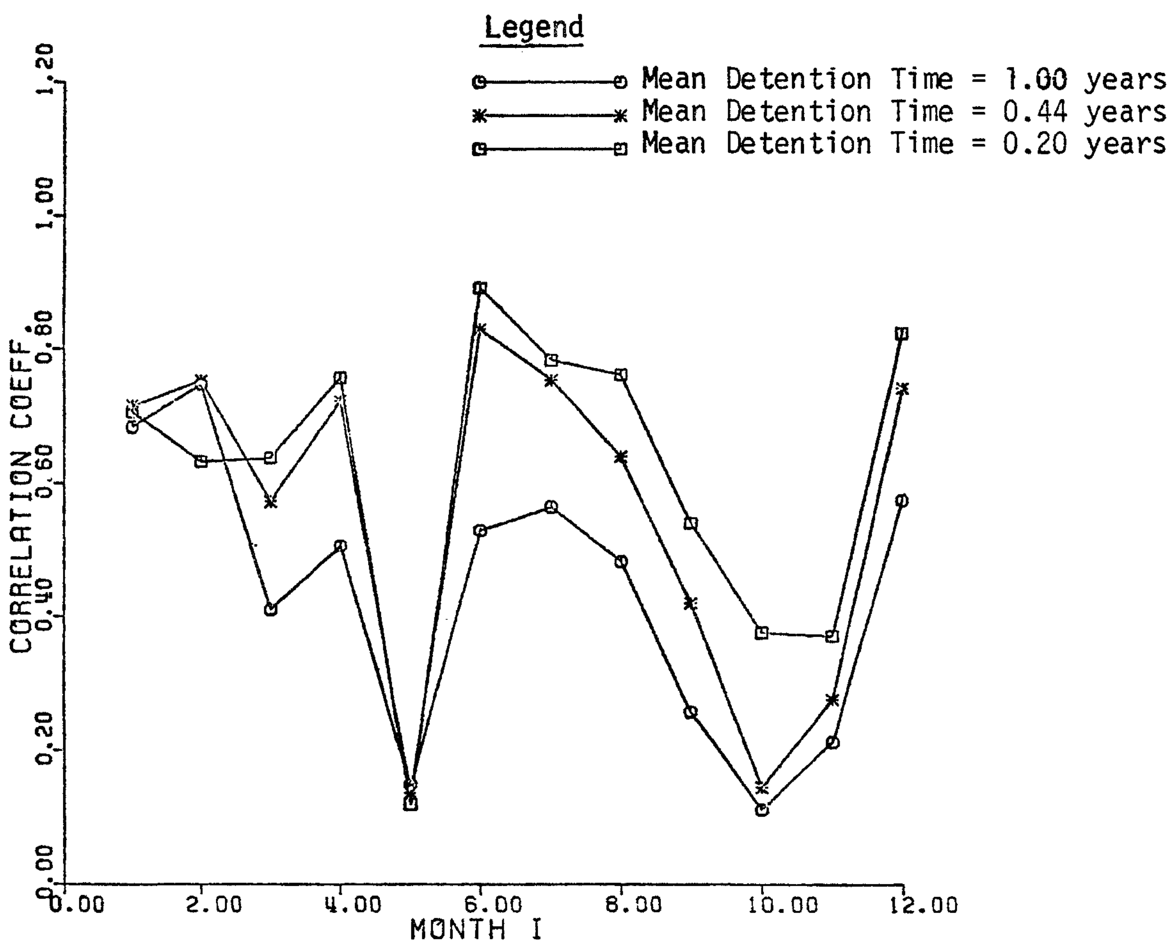


Figure 8.3 (b): Monthly Trends in the Correlation Coefficient of Optimal Release and Previous Period Inflow.

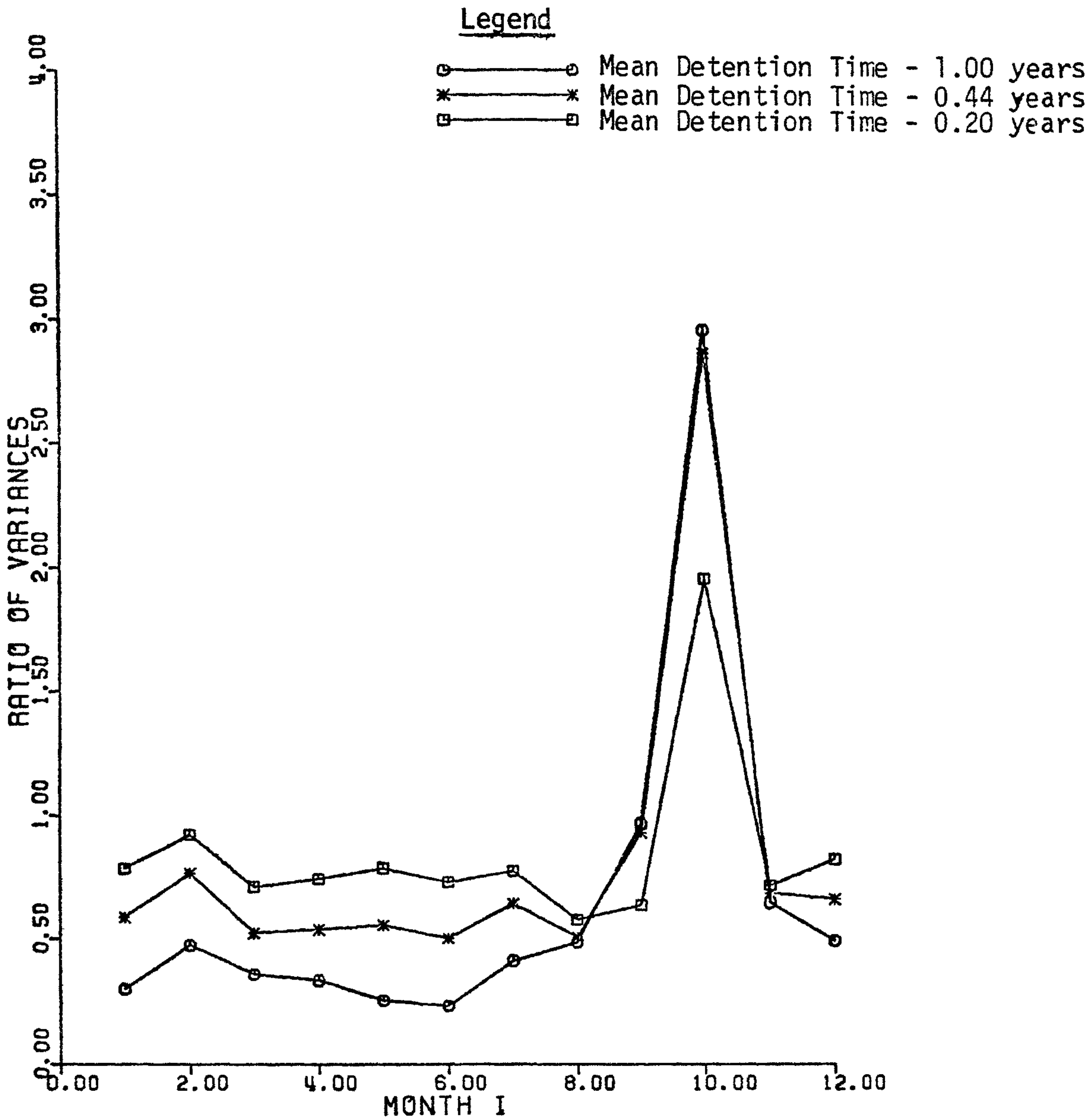


Figure 8.4 (a): Monthly Trends in the Ratio of Variances of Optimal Release and Current Inflow.

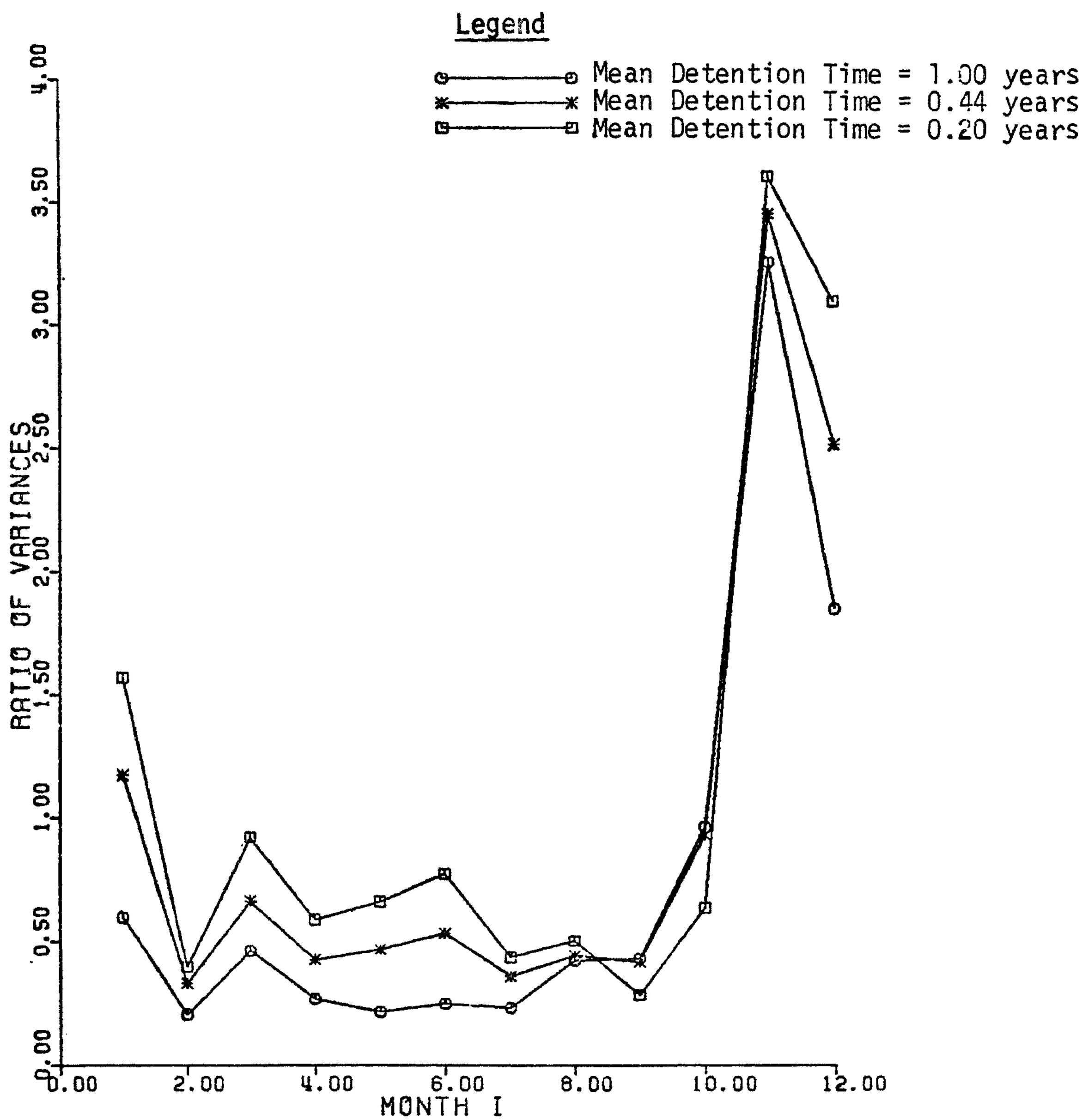


Figure 8.4 (b): Monthly Trends in the Ratio of Variances of Optimal Release and Previous Period Inflow.

In this section this information will be used in a attempt to predict the value of the regression coefficient, B_1, for an arbitrary reservoir site.

Figures 8.3 and 8.4 illustrate the trends in the correlation coefficient and ratio of variances with changes in the reservoir mean detention time. Since the dynamic programming policies, DP1 and DP2, derived under a two-sided quadratic loss function are target independent, it is expected that the regression coefficient B_1, will depend only on the relationship of reservoir capacity to the mean annual inflow (mean detention time). Consequently, the relationships shown in Figures 8.3 and 8.4 can be used in conjunction with linear extrapolation to predict the regression coefficient, B_1, for any general reservoir site. Thus, optimal release policies of the DP1 and DP2 type may be obtained directly, without solving the dynamic programming algorithm.

The trends in Figures 8.3 and 8.4 indicate that:

a) The correlation of monthly releases with the current or previous period's inflow decreases with increasing mean detention time.

b) Except in the low flow months of September and October, the ratio of variance of release to the variance of current or previous period's inflow, decreases with an increase in the mean detention time.

CHAPTER IX

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

In the previous three Chapters, monthly release policies are derived for a single, multi-purpose reservoir using Hoover Reservoir, located in Central Ohio, as a case-example. Chapters 6 and 7 illustrate the application of the chance-constrained linear programming and dynamic programming-regression methodologies, respectively. A comparison of monthly release policies using these two mathematical optimization techniques, and the search for an appropriate form of a monthly release policy is the major emphasis of Chapter 8. Simulation procedures, in conjunction with operational hydrology, are extensively used to measure and verify the performance of the monthly release policies. Chapter 3 and 4 are devoted to the study of the existing design and operation of Hoover Reservoir.

Based on the results presented in this study the following conclusions are made:

1) Statistical analysis of releases from Hoover Reservoir, since its operation in 1954, indicates that the release policy followed in the past is close to the standard policy. The safe-yield from the reservoir is about 68 M.G.D.
2) For the original form of the chance-constrained linear programming release policies (Equations 6.1 and 6.4), the simulation estimates of reliability in satisfying the chance-constraints on the maximum and minimum storages

and releases, are well within the selected levels of reliability at all target levels. It is also observed that although the levels of reliability were set equal in all months, simulation indicates a varying degree of reliability between months.

3) Statistics on the average loss per year suggest that the optimal policies derived using the chance-constrained linear programming approach have losses that are very similar to those incurred under the standard policy (see Table 6.8). However, comparison of reliability of meeting similar draft-levels suggests that the standard policies gives higher reliabilities than the chance-constrained programming policies.

4) Since the transformed chance-constrained linear policies (Equations 6.2 and 6.5) are merely an algebraic revision of their original counterparts (Equations 6.1 and 6.3), it is reasonable to expect that their performance would be identical, provided continuity in the operation of the reservoir is maintained between successive months. However, for a finite reservoir capacity, discontinuities due to maximum and minimum storage violations and prediction of negative releases (set equal to zero in this study), cause discrepancies in the performance of the transformed and original forms. Consequently, it is

recommended that the chance-constrained programming policies, particularly LDR1, be used in their original form to achieve the lower average losses associated with this form.

5) The adequacy of the chance-constrained programming approach to derive optimal monthly release policies is restricted by the relationship between the minimum guaranteed flow, q_{min}, and the reliability, α', imposed on meeting this flow.[1] (refer to Figure 5.2). For instance, results show that the chance-constrained programming policies meet the safe-yield of 68 M.G.D.[2] with a reliability of 54%, while at a similar draft the standard policy gives a higher reliability of 98%.

Analysis of optimal release policies obtained using the dynamic programming-regression approach indicates:

1) Simple correlations of release with selected independent variables are useful in a preliminary selection of the form of linear and non-linear policies. Also, such coefficients may assist in the understanding of the physical behavior of the system under consideration.

2) Inflows in previous periods become more important in predicting releases in any given period on the mean detention time increases. This is expected, since a

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1. This relationship is characteristic of the reservoir site.
2. Safe-yield of Hoover Reservoir.

higher mean detention time implies a reservoir with a larger capacity, allowing releases in any period to be more dependent on past inflows through increased reservoir regulation.

3) Using the maximum R^2 criterion, linear monthly policies, M1, are generally as good as, or better than, non-linear policies, M2 and M3, for the two-sided quadratic loss function. Results suggest that this is true at all levels of reservoir mean detention times. Conversely, non-linear policies are more appropriate than linear policies of the one-sided quadratic loss function.

4) The maximum R^2 criterion does not always produce that best operational model as measured by simulation results based upon the average total loss per year. Thus, simple policies with a lower R^2 may be more appropriate than policies with higher R^2. The present study indicates that this is the case for non-linear policies but not for linear policies.

5) Although the release policies, both linear and non-linear, are target independent under the two-sided quadratic loss function, there is a particular target level at which the average loss per year is a minimum. This level is close to the mean annual inflow. Release policies derived using a one-sided quadratic loss function are target dependent.

6) For the two-sided quadratic loss function, the complete linear model M1 is the best policy. The standard policy is not optimal except at a target equal to the mean annual inflow.

7) The percentage improvement in the performance of the dynamic programming policies, derived under a two-sided quadratic loss function, over the standard policy increases with an increase in the mean detention time. This is undoubtedly due to the nature of these policies, since the standard policy incorporates only the current inflow and storage in making release decisions at all levels of the mean detention time. In contrast, the dynamic programming policies allow the incorporation of more of the history of the system, by reflecting the increased dependence of current releases on past inflows at higher mean detention times.

8) Under a one-sided quadratic loss function, the standard policy is optimal at a target level of 75 MGD. This is the current draft at Hoover Reservoir and closely represents the safe-yield.* At higher targets, non-linear policies, M2 and M3, are better than either the standard policy or linear policy M1. For targets lower than 75 MGD, the standard policy remains favorable.

9) The difference in the overall performance of the simple one-variable policies, DP1 and DP2, is insignificant when

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* Includes evaporation loss

compared with policies DP3 and the complete linear model M1.

In the comparison of chance-constrained programming policies, LDR1 and LDR2, with the dynamic programming policies DP1 and DP2, mean detention times greater than 0.44 years (existing Hoover Reservoir design) are not emphasized in the present study. This is justified since at mean detention times greater than 0.44 years, the reliability, α', in the chance-constrained programming approach, would be below 0.50, which is unrealistic from a physical point of view. However, in order to examine, separately, the performance of the dynamic programming policies at safe-yield drafts, mean detention time greater than 0.44 years are included.

Based on the results in Chapter 8, the following conclusions are made:

1) Unlike the chance-constained linear programming policies, LDR1 and LDR2, the coefficient associated with the current or previous period's inflow in the dynamic programming policies, DP1 and DP2 (Equations 8.3 and 8.4) is not equal to unity. This is true at all target levels and mean detention times.

2) Simulations under the minimum guaranteed flow, q_i, and the safe-yield drafts (for mean detention times less than 0.44 years) indicate, that, while the tested chance-constrained programming policies satisfy the reliability,

α', of meeting these drafts, the dynamic programming policies fail to do so (particularly in the low flow months). The reservoir capacity is maintained at the same level for both types of policies.

3) For the dynamic programming policies corresponding to mean detention times less than 0.44 years, the performance in terms of reliability, α', deteriorates with decreasing mean detention time in the low flow months (August-December), while performance improves in the high flow months (January-April).

4) At a mean detention time of 0.44 years (existing Hoover Reservoir design) both the tested chance-constrained programming and dynamic programming policies satisfy the reliability, α', of meeting the minimum guaranteed flow, q_i, and the safe-yield, respectively.

5) Examining the performance of dynamic programming policies over the range of mean detention times, 0.20-1.0 years, it is observed that the reliability of meeting safe-yield drafts deteriorates with increasing mean detention times.

6) At a detention time of 0.44 years the derived dynamic programming policies perform more adequately than either LDR1 or LDR2 at a target level of 75 MGD, e.g., the dynamic programming policies produce lower losses

while achieving at least as high reliability levels. This observation would argue for more research on deriving generalized linear decision rules wherein the present and past period's inflows are weighted in an optimal manner.

Recommendations for Future Research: Based on the findings in this study, it is recommended that future research be directed towards the following considerations.

1) Test the sensitivity of all models to the input hydrology, in the following ways:

   a) Investigate different probability distributions of monthly flows and their effect on simulation studies. For example, the log-Pearson type III distribution has been used by the Corps of Engineers in their HEC-4 computer program, Monthly Streamflow Simulation. The log-Pearson type III distribution is a special case of the Gamma distribution. The latter distribution gave a good fit to the historical streamflow data in the present study.

   b) Examine the sensitivity of results to the estimates of mean, variance and serial correlation in the log-normal model used in this study.

   c) Evaluate the extent to which changes in the watershed conditions over time could be expected to influence the primary statistics of streamflows as determined in this study. This could be achieved through the use of models such as Stanford Watershed Model, existing long-term meteorological records at Columbus, and assumptions concerning expected changes in the characteristics of the watershed.

2) The use of the general form of the linear decision rule, $x_t = \lambda_o S_t + \lambda_1 R_t + \lambda_2 R_{t-1} t \ldots \lambda_n R_{t-n-1} - b$, requires the evaluations of the optimal values of the coefficients, λ_1 , $\lambda_2 \ldots \lambda_n$. Consequently, there is a need for developing methods to determine these coefficients explicitly, prior to solving the chance-constrained model. The dynamic programming-regression approach discussed in Chapter 8 may provide valuable information to assist in the search for these optimal coefficients.

3) Develop a dynamic programming algorithm which would incorporate reliability considerations either explicitly or implicitly to derive optimal reservoir operating policies.

APPENDIX A

Seepage Computations

Reservoir Seepage: Seepage into or out of a reservoir may be obtained using Darcy's equation:

$$q = K(dh/dx)w$$

where,

q = flow (gpd/ft)

dh = potential drop (ft.)

dx = distance over which the potential drop is observed (ft.)

w = width of seepage face (ft.)

K = permeability of the soil (gpd/ft^2)

Computational details at various sections (Figure A-1) are shown in Table A-1. A negative flow value indicates that the seepage is out of the reservoir. The head loss, dh_1, is the difference in the piezometric heads between a representative point in the bank of the reservoir, where the head is known, and the bottom of the reservoir. The water level in the reservoir is at 890 feet above mean sea level. Similarly, head loss dh_2 is the potential drop between the same points when the reservoir is empty. However in the latter case the bottom of the reservoir is no longer an equipotential line but is a free surface. Since the ratio of the width of the reservoir to its depth is large along these sections it is reasonable to assume a constant head along the free surface. The head under these conditions will be equal to the elevation of the bottom of the reservoir above mean sea

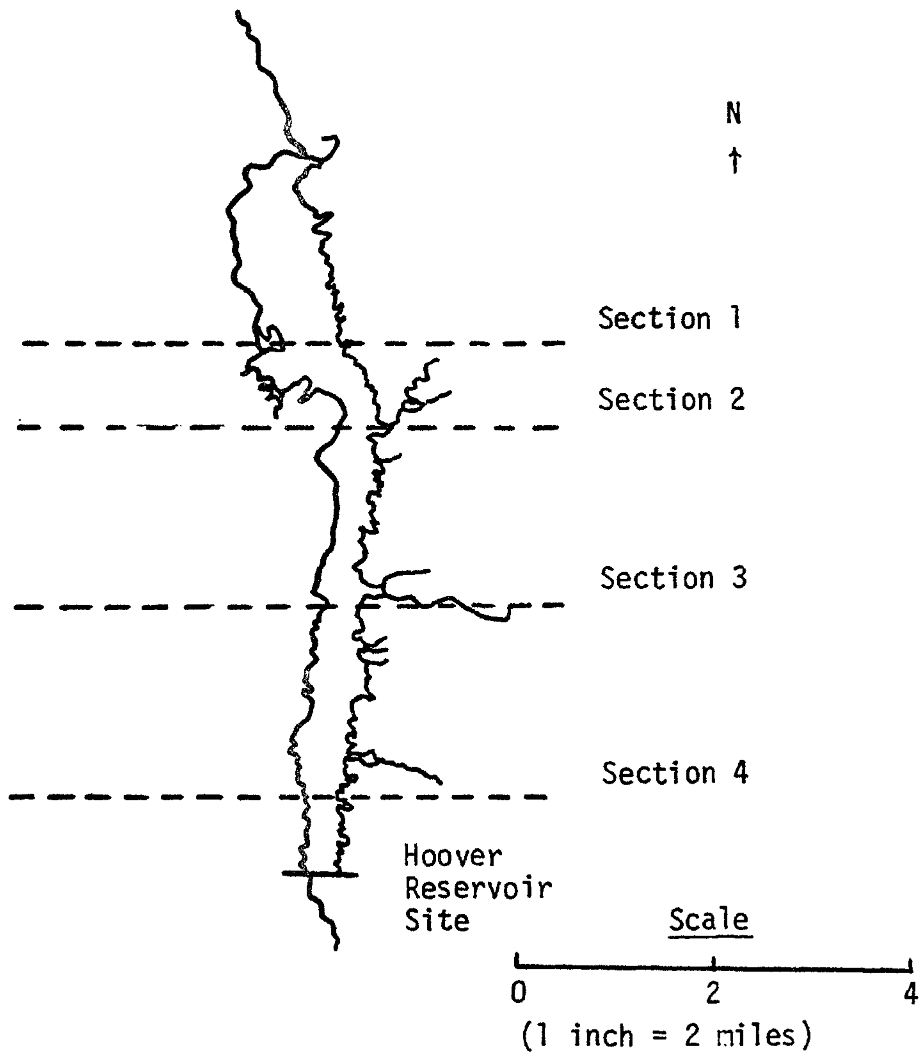


Figure A-1 Hoover Reservoir with the Representative Cross Sections

Table A-1: Seepage Computation

Section	Section Length (miles)	Bank	Soil Type	w (ft)	dh_1* (ft)	dh_2** (ft)	dx (ft)	K*** (gpd/ft^2)	q_1* (gals/day/ft)	q_2** (gals/day/ft)
1	2.0	West	Clayey till	2112	10.0	30.0	1584	0.10	1.33	4.00
	1.6	East	Clayey till	1764	10.0	30.0	1232	0.10	1.43	4.30
2	1.6	West	Clayey till	2112	-5.0	5.0	1320	0.10	-0.80	0.80
	2.0	East	Clayey till	1056	-7.5	2.5	1232	0.10	-0.64	0.21
3	1.8	West	Clayey till	704	10.0	47.5	1320	0.10	0.53	2.53
	1.6	East	Clayey till	880	0.0	37.5	352	0.10	0.0	9.38
4	1.6	West	Shale	420	1.0	41.0	1056	0.001	0.0004	0.016
	1.8	East	Shale	2112	50.0	100.0	1020	0.001	0.104	0.208

*Reservoir storage level at a pool elevation of 890 M.S.L.

**Reservoir empty

***Representative values taken from ground water maps published by the Ohio Department of Natural Resources, Division of Water.

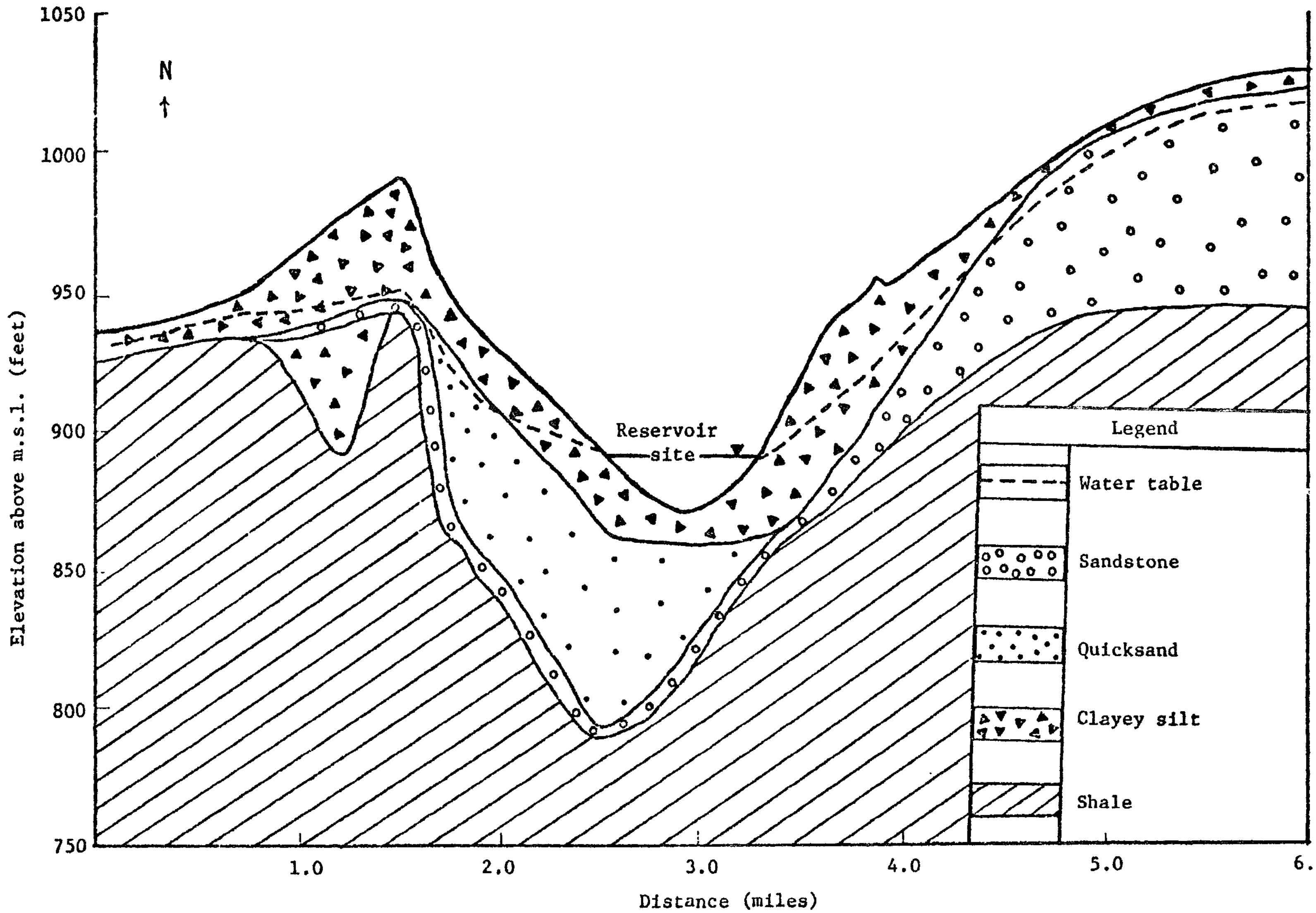


Figure A-2: Geologic Profile at Section 1

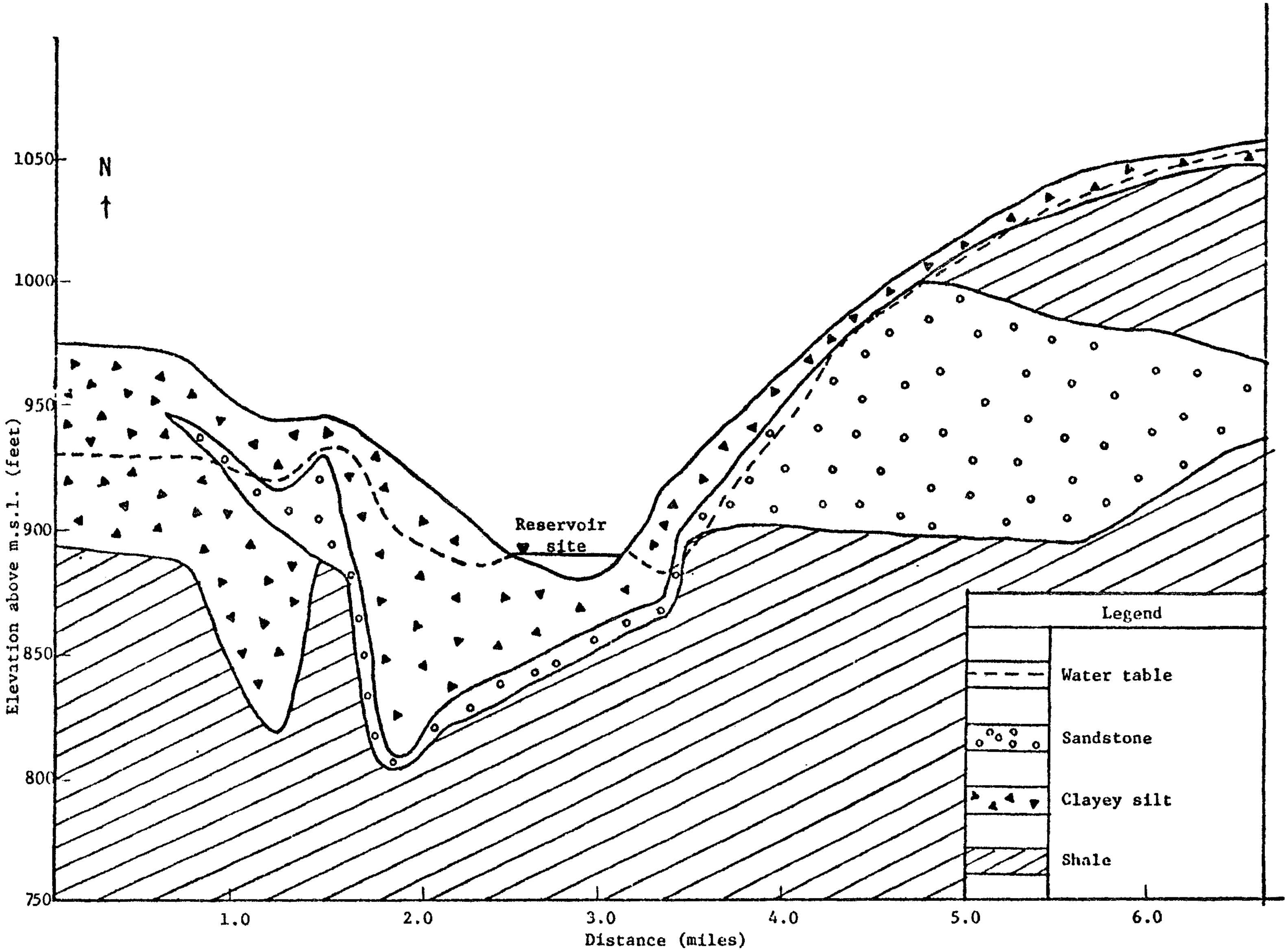


Figure A-3: Geologic Profile at Section 2

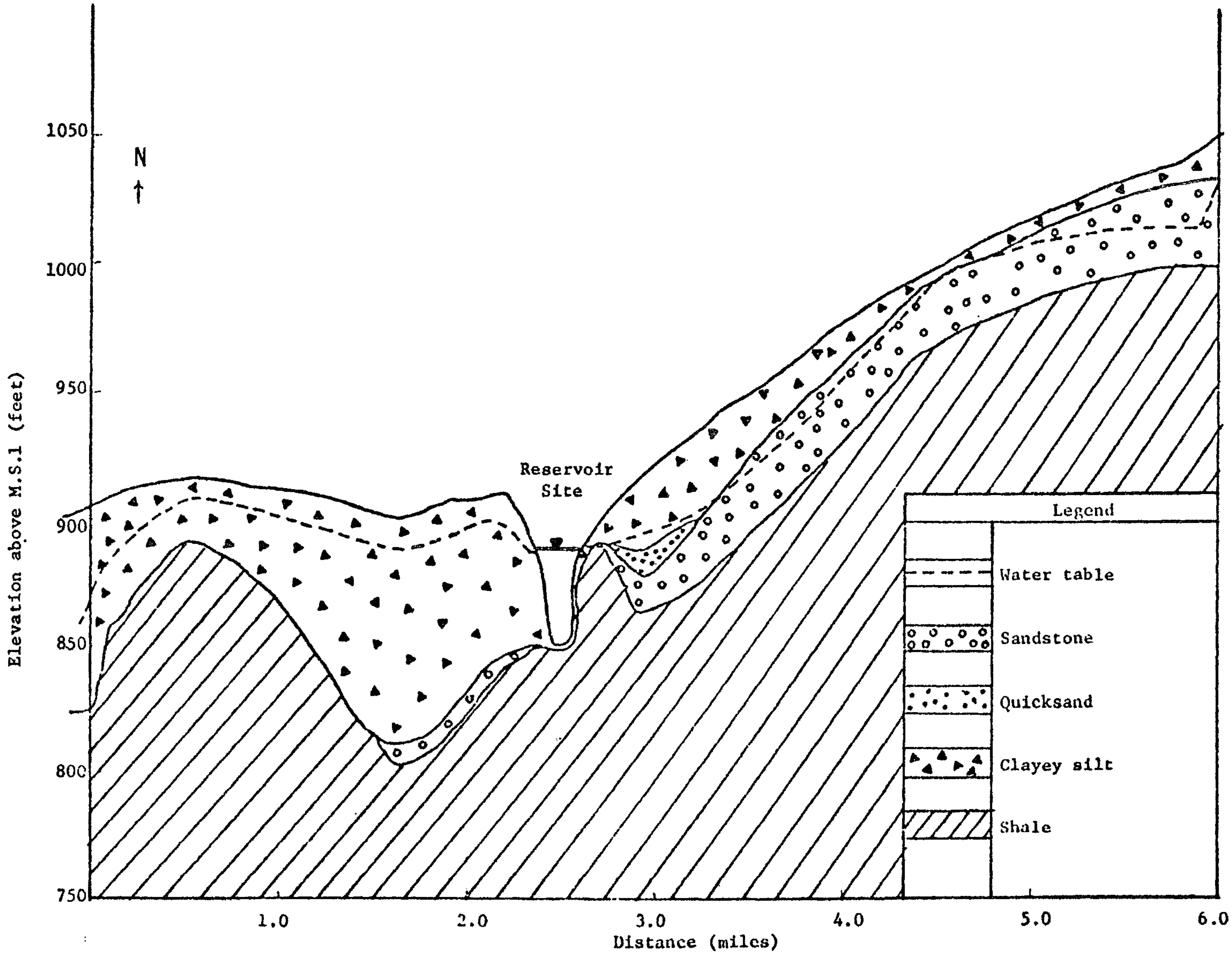


Figure A-4: Geologic Profile at Section 3

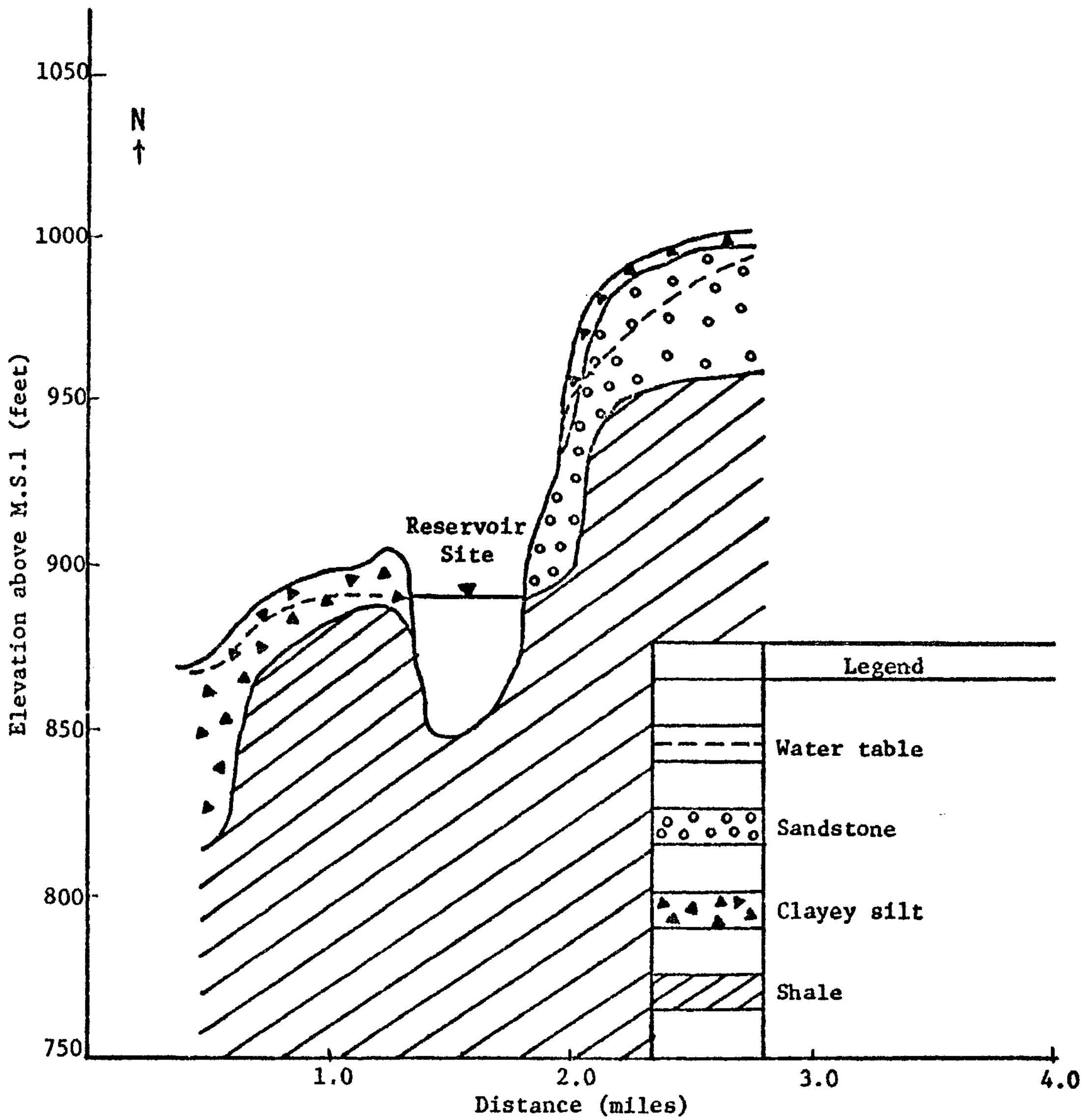


Figure A-5: Geologic Profile at Section 4

level. Figures A-2 through A-5 show the geologic profiles of representative cross sections of the reservoir. These profiles have been obtained using well-log data. The section lengths associated with each of the cross-sections represents the distance in the longitudinal direction over which the cross-section remains geologically similar. Net seepage into the reservoir is shown in Table A-2.

Table A-2: Net Seepage

Bank		Net Seepage in M.G.D. Pool level at Elevation 890 ft. above M.S.L.	Net Seepage in M.G.D. Reservoir Empty
West		0.012	0.073
East		0.006	0.120
	Total	0.018	0.193

APPENDIX B

Evaporation Computations

Average Monthly Lake Evaporation: The average lake evaporation at Columbus is derived using available data from Coshocton, as follows:

a) The average monthly lake evaporation is computed using available data at Coshocton, over the period 1939-1975;

b) The observed annual lake evaporation for Coshocton and Columbus are obtained from the Hydrologic Atlas. (Eagon, 1962) These values are:

Annual lake evaporation for Coshocton = 32.30 inches.

Annual lake evaporation for Columbus = 33.50 inches.

Increase in evaporation at Columbus = 1.20 inches.

c) The increase in the annual lake evaporation at Columbus of 1.20 inches, as computed in Step b, is distributed between the months using the following weights, w_i:

$$w_i = \frac{\text{Average lake evaporation in month i at Coshocton}}{\text{Annual lake evaporation at Coshocton}}$$

Detailed computations are illustrated in Table B-1.

Table B-1: Average Monthly Lake Evaporation at Columbus

Month	Average Monthly Evaporation at Coshocton (inches)	Weight W_i	Increase in Average Monthly Evaporation (inches)	Average Monthly Evaporation at Columbus (inches)
January	0.601	0.01858	0.023	0.624
February	0.876	0.02708	0.033	0.909
March	1.727	0.05340	0.064	1.791
April	2.977	0.09204	0.110	3.087
May	4.297	0.13285	0.159	4.456
June	5.042	0.15589	0.187	5.229
July	5.331	0.16482	0.198	5.529
August	4.607	0.14244	0.171	4.778
September	3.240	0.10017	0.120	3.360
October	2.003	0.06329	0.076	2.079
November	1.007	0.03113	0.037	1.044
December	0.592	0.01830	0.022	0.614
Sum	32.30	1.00	1.20	33.50

APPENDIX C

Summary of Statistical Tests of Significance Between Computed and Natural Inflows

Table C-1: Results of Statistical Significance Tests

Month	F Value	2-Tail* Prob.	Pooled Variance Estimate: T Value	Pooled Variance Estimate: Degrees of Freedom	Pooled Variance Estimate: 2-Tail* Prob.	Separate Variance Estimate: T Value	Separate Variance Estimate: Degrees of Freedom	Separate Variance Estimate: 2-Tail* Prob.
January	2.63	0.058	0.91	32	0.371	0.89	24.39	0.386
February	1.42	0.486	1.44	32	0.160	1.42	29.49	0.165
March	1.72	0.295	-0.33	32	0.745	-0.33	31.31	0.741
April	2.07	0.163	-0.85	32	0.403	-0.87	30.34	0.394
May	2.13	0.149	-2.13	32	0.041	-2.17	30.18	0.038
June	1.34	0.561	0.53	32	0.601	0.52	29.92	0.504
July	1.62	0.353	-0.27	32	0.789	-0.27	31.56	0.786
August	4.43	0.005	-0.24	32	0.813	-0.23	20.89	0.820
September	1.59	0.356	-0.68	32	0.500	-0.67	28.57	0.506
October	8.34	0.000	-0.75	32	0.458	-0.79	21.45	0.440
November	5.00	0.004	-0.67	32	0.505	-0.70	24.13	0.489
December	1.36	0.538	0.26	32	0.797	0.26	29.79	0.799

* This is the probability of finding a sample which is statically better.

APPENDIX D

Probability Distribution Analysis

Appendix D-1: Relating parameters of normal and log-normal distributions for single and bi-variate cases.

The following are the main objectives of the discussion in this Appendix:

1) Application of the change of variable technique (Hogg and Craig, 1970) to derive:

   a) the probability density function of x given the random variable $y \sim N\ (\mu_y, \sigma_y^{\ 2})$ and the transformation $y = \log_e x$.

   b) the joint probability density function of random variables x_1 and x_2 given that random variables y_1 and y_2 are $N(\mu_{y1}, \sigma_{y1}^2)$ and $N(\ \mu_{y2}, \sigma_{y2}^2)$ respectively. Also $f(y_1, y_2)$ is a bivariate normal distribution and,

$$y_1 = \log_e x_1$$

$$y_2 = \log_e x_2$$

2) To obtain the relationships between the parameters:

   a) $\mu_x, \sigma_x^{\ 2}, \rho_x$ and $\mu_y, \sigma_y^{\ 2}, \rho_y$

   b) $\mu_{x1}, \mu_{x2}, \sigma_{x1}^2, \sigma_{x2}^2$ and $\mu_{y1}, \sigma_{y1}^2, \mu_{y2}, \sigma_{y2}^2$

   c) $\rho_{x1,x2}$ and $\rho_{y1,y2}$

where, μ, σ^2, and ρ are means, variances and correlation coefficients of the indexed random variables, respectively.

<u>Probability density function of random variable x given the distribution of random variable y, where $y = \log_e x$.</u>

1. <u>Single-variate case</u> - Let the random variable, y, be normally distributed with mean, μ_y, and variance σ_y^2. The density function can be represented by,

$$\left\{ f(y) = \frac{1}{\sigma_y\sqrt{2\pi}} e^{-(y - \mu_y)^2/2\sigma_y^2}; \ -\infty \leq y \leq \infty; \right\} \quad (1)$$

Given the transformation, $y = \log_e x$ the probability density function of x is defined by the change of variable technique as (Hogg and Craig, 1970):

$$\left\{ g(x) = f(y) \left[J \right] \right\} \quad (2)$$

where,

the Jacobian, $J = \left[\frac{dy}{dx}\right]$, (3)

and for $y = \log_e x$, $J = \left[\frac{1}{x}\right]$

Using equations 1, 2 and 3, the probability function of random variable x may be expressed as:

$$\left\{ g(x) = \frac{1}{x\sigma_y\ 2\pi} e^{-(\log_e x - \mu_y)^2/2\sigma_y^2} \quad 0 < x \leq \infty \right\} \quad (4)$$

where, g(x) is called the log-normal distribution.

Since, $y \sim N(\mu_y, \sigma_y^2)$, the moment generating function may be written as:

$$\left\{ M(t) = E(e^{t \cdot y}) = \int_{-\infty}^{\infty} e^{t \cdot y} \cdot f(y)\, dy = e^{(\mu_y t + \frac{1}{2} \sigma_y^2 t^2)} \right\} \quad (5)$$

where, t = a parameter.

For the log-normally distributed random variable x, a general expression for the t^{th} moment may be obtained using equations 2, 3 and 5. Thus,

$$E(x^t) = \int_{-\infty}^{\infty} x^t \cdot f(x) \cdot dx$$

$$= \int_{-\infty}^{\infty} e^{t \cdot y} \cdot f(y) \cdot dy$$

$$= e^{(\mu_y t + \frac{1}{2} \sigma_y^2 t^2)}$$

Thus, $$\left\{ E(x^t) = M(t) = e^{(\mu_y t + \frac{1}{2} \sigma_y^2 t^2)} \right\} \quad (6)$$

To get any moment of the random variable, x, the parameter, t, in Equation 6 is assigned an integer value greater than zero. Hence,

$$\mu_x = E(x) = e^{(\mu_y + \frac{1}{2} \sigma_y^2)} \quad \text{for } t = 1 \quad (7)$$

$$E(x^2) = e^{2(\mu_y + \sigma_y^2)} \quad \text{for } t = 2$$

and, $$\sigma_x^2 = E(x^2) - (E(x))^2 \quad (8)$$

$$e^{2(\mu_y + \sigma_y^2)} - e^{2(\mu_y + \sigma_y^2/2)}$$

Equations (7) and (8) relate the mean and variances of the random variables x and y respectively. The variance, σ_x^2, can be alternatively expressed as,

$$\sigma_x^2 = e^{(2\mu_y + \sigma_y^2)}\left[e^{\sigma_y^2} - 1\right] \tag{9}$$

or,

$$\sigma_x^2 = \mu_x^2 \left[e^{\sigma_y^2} - 1\right]$$

where,

$$\mu_x = e^{(\mu_y + \sigma_y^2/2)}$$

<u>Expression for the skewness coefficient, γ_x, of the log-normal random variable x.</u>

$$E\left[x - \mu_x\right]^3 = E\left[x^3 - 3x^2 \mu_x + 3x\mu_x^2 - \mu_x^3\right]$$

$$= E(x^3) - 3\mu_x E(x^2) + 3E(x)\mu_x^2 - \mu_x^3$$

$$= E(x^3) - 3\mu_x E(x^2) + 2\mu_x^3$$

From Equation 6,

$$E(x^3) = e^{(3\mu_y + 9/2\sigma_y^2)} = e^{3(\mu_y + 3/2\sigma_y^2)}$$

and,

$$-3\mu_x E(x^2) = -3e^{(\mu_y + \frac{1}{2}\sigma_y^2)}\; e^{(2\mu_y + 2\sigma_y^2)}$$

$$= -3e^{(3\mu_y + 5/2\sigma_y^2)}$$

$$2\mu_x^3 = 2\, e^{3(\mu_y + \sigma_y^2/2)}$$

Therefore,

$$E(x - \mu_x)^3 = e^{3(\mu_y + 3/2\sigma_y^2)} - 3e^{(3\mu_y + 5/2\sigma_y^2)} + 2e^{3(\mu_y + \sigma_y^2/2)}$$

$$= e^{3(\mu_y + \sigma_y^2/2)} \left\{ e^{3\sigma_y^2} - 3e^{\sigma_y^2} + 2 \right\}$$

$$= \mu_x^3 \left\{ e^{3\sigma_y^2} - 3e^{\sigma_y^2} + 2 \right\}$$

Hence, the skewness coefficient is

$$\gamma_x = \frac{E\left[x - \mu_x\right]^3}{\sigma_x^3} = \frac{\mu_x^3 (e^{3\sigma_y^2} - 3e^{\sigma_y^2} + 2)}{\mu_x^3 (e^{\sigma_y^2} - 1)^{3/2}}$$

or,

$$\gamma_x = \left\{ (e^{3\sigma_y^2} - 3e^{\sigma_y^2} + 2) \Big/ (e^{\sigma_y^2} - 1)^{3/2} \right\} \tag{10}$$

2. <u>Bivariate Case</u> - Given the random variables y_1 and y_2 normally distributed with means, μ_{y1} and μ_{y2}, and variances, σ_{y1}^2 and σ_{y2}^2, respectively, the probability density function of their joint distribution may be expressed as

$$f(y_1, y_2) = \frac{1}{2\pi\sigma_{y1} \cdot \sigma_{y2}\sqrt{1 - \rho_{y1,\,y2}^2}}\, e^{-q/2}; \quad \begin{array}{l} -\infty \leq y_1 \leq \infty \\ -\infty \leq y_2 \leq \infty \end{array} \tag{11}$$

where, $q = \dfrac{1}{1 - \rho_{y1,\,y2}^2} \left[\left(\dfrac{y_1 - \mu_{y1}}{\sigma_{y1}}\right)^2 - 2\rho_{y1,\,y2} \left(\dfrac{y_1 - \mu_{y1}}{\sigma_{y1}}\right)\left(\dfrac{y_2 - \mu_{y2}}{\sigma_{y2}}\right) + \left(\dfrac{y_2 - \mu_{y2}}{\sigma_{y2}}\right)^2 \right]$

Let, $y_1 = \log_e x_1$ or $x_1 = e^{y_1}$

and $y_2 = \log_e x_2$ or $x_2 = e^{y_2}$ (12)

Then by the change of variable technique, the joint probability density function of random variables x_1 and x_2 may be derived as follows:

$$g(x_1, x_2) = f(y_1, y_2) \cdot \left[J \right] \tag{13}$$

where, $f(y_1, y_2)$ is the joint probability density function of y_1 and y_2 (Equation 9) and the Jacobian $J = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{x_1} & 0 \\ 0 & \frac{1}{x_2} \end{bmatrix} = \frac{1}{x_i x_2}$$

Thus,

$$g(x_1, x_2) = \frac{1}{2\pi\, \sigma_{y1} \cdot \sigma_{y2} \sqrt{(1-\rho^2_{y1,\, y2})}} \cdot \frac{1}{x_1 x_2}\, e^{-q^1/2}; \tag{14}$$

$$0 < x_1 \leq \infty$$
$$0 < x_2 \leq \infty$$

where,

$$q^1 = \frac{1}{1 - \rho^2_{y1,\, y2}} \left[\left(\frac{\log_e x_1 - \mu_{y1}}{\sigma_{y1}} \right)^2 - 2\rho_{y1,\, y2} \left(\frac{\log_e x_1 - \mu_{y1}}{\sigma_{y1}} \right) \left(\frac{\log_e x_2 - \mu_{y2}}{\sigma_{y2}} \right) + \left((\log_e x_2 - \mu_{y2}/\sigma_{y2}) \right)^2 \right]$$

The moment generating function for the bivariate normal distribution function $f(y_1, y_2)$ is,

$$M(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1 y_1 + t_2 y_2} \cdot f(y_1, y_2) \cdot dy_1 \cdot dy_2 \qquad (15)$$

where, t_1, t_2 are parameters.

Using the above equation, the general moment for the joint probability density function $g(x_1, x_2)$ can be written as

$$E\left[x_1^{t_1} \cdot x_2^{t_2}\right] = \int_{0}^{\infty}\int_{0}^{\infty} x_1^{t_1} \cdot x_2^{t_2} \cdot g(x_1, x_2) dx_1 \cdot dx_2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1 y_1} \cdot e^{t_2 y_2} \cdot f(y_1, y_2) dy_1 \cdot dy_2 \qquad (16)$$

Hence, similar to the single variate case, Equations 13 and 14 are identical. Thus,

$$E\left[x_1^{t_1} \cdot x_2^{t_2}\right] = M(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1 y_1} \cdot e^{t_2 y_2} \cdot f(y_1, y_2) \cdot dy_1 \cdot dy_2 \qquad (17)$$

The moment generating function, $M(t_1, t_2)$, for the bivariate normal distribution $f(y_1, y_2)$ can be written in its final form as:

$$M(t_1, t_2) = \left\{ \exp \quad \mu_{y1}t_1 + \mu_{y2}t_2 + \frac{\sigma_{y1}{}^2 t_1{}^2 + 2\,\rho_{y1,\,y2} \cdot \sigma_{y1} \cdot \sigma_{y2} \cdot t_1 \cdot t_2}{2} \right\}$$

where, exp = exponential function base e. (18)

Substituting positive integer values for the parameters t_1, t_2 in the above equations yields the desired moments of the joint probability density function $g(x_1, x_2)$. Hence,

$$\mu_{x1} = E\left[x_1\right] = \exp\left[\mu_{y1} + \sigma_{y1}^2/2\right] \quad \text{for } t_1 = 1,\ t_2 = 0 \tag{19}$$

$$\mu_{x2} = E\left[x_2\right] = \exp\left[\mu_{y2} + \sigma_{y2}^2/2\right] \quad \text{for } t_1 = 0,\ t_2 = 1$$

$$E(x_1^2) = \exp 2\left[\mu_{y1} + \sigma_{y1}^2\right] \quad \text{for } t_1 = 2,\ t_2 = 0$$

$$E(x_2^2) = \exp 2\left[\mu_{y2} + \sigma_{y2}^2\right] \quad \text{for } t_1 = 0,\ t_2 = 2$$

$$E(x_1 \cdot x_2) = \exp(\mu_{y1} + \mu_{y2} + \sigma_{y1}^2/2 + \rho_{y1,y2} \cdot \sigma_{y1} \cdot \sigma_{y2} + \sigma_{y2}^2/2)$$

$$\text{for } t_1 = 1,\ t_2 = 1$$

$$\sigma_{x1}^2 = E\,(x_1^2) - E(x_1)^2 = \exp 2(\mu_{y1} + \sigma_{y1}^2) - \exp 2(\mu_{y1} + \sigma_{y1}^2/2)$$

$$\sigma_{y2}^2 = E\,(x_2^2) - E(x_2)^2 = \exp 2(\mu_{y2} + \sigma_{y2}^2) - \exp 2(\mu_{y2} + \sigma_{y2}^2/2)$$

Auto-Correlation Coefficient, $\rho_{X1,\ x2}$

The pearson product moment correlation or the auto-correlation coefficient is defined as,

$$\rho_{x1,\ x2} = \left\{ \frac{E(x_1 \cdot x_2) - \mu_{x1} \cdot \mu_{x2}}{\sigma_{y1} \cdot \sigma_{y2}} \right\} \tag{20}$$

Using the expressions shown in Equation (19) the above equation may be simplified to give

$$\rho_{x1,\ x2} = \left\{ \frac{e^{(\rho_{y1,\ y2} \cdot \sigma_{y1} \cdot \sigma_{y2})} - 1}{(e^{\sigma_1^2} - 1)^{\frac{1}{2}} \cdot (e^{\sigma_{y2}^2} - 1)^{\frac{1}{2}}} \right\} \tag{21}$$

where, $\rho_{y1,\ y2}$ is the auto-correlation coefficient of the normally distributed random variables y_1 and y_2.

Appendix D-2: Maximum Likelihood Estimates of Parameters.

a) Normal Distribution

If the random variable, x, is normally distributed with mean μ_x and variance σ_x^2 then the probability density function of x may be represented by:

$$f(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-(x - \mu_x)^2/2\sigma_x^2} \qquad -\infty \leq x \leq \infty \tag{1}$$

The maximum likelihood function is defined as,

$$L(x, \theta) = \prod_{i=1}^{n} f(x_i, \theta)$$

$$L(x, \mu_x, \sigma_x) = \left(\frac{1}{\sigma_x\sqrt{2\pi}}\right)^n e^{-\frac{1}{2\sigma x^2}\sum_{i=1}^{n}(x_i - \mu_x)^2} \tag{2}$$

where, x_i = i^{th} random sample drawn from the population with the probability density function $f(x, \theta)$.

The parameters μ_x and σ_x^2 may be estimated as follows:

Set, $$\frac{\partial}{\partial\mu_x}(\log_e L) = 0 \tag{3}$$

and, $$\frac{\partial}{\partial\sigma_x}(\log_e L) = 0 \tag{4}$$

The natural log of the likelihood function, $\log_e L$, as defined in Equation (2) is:

$$\left\{ \log_e L(x, \mu_x, \sigma_x) = -n \log_e(\sigma_x) - n \log_e(2\pi) - \frac{1}{2\sigma_x^2} \right\} \sum_{i=1}^{n} (x_i - \mu_x)^2 \qquad (5)$$

Differentiating the above equation with respect to μ_x and setting it equal to zero as in Equation (3) yields,

$$\left\{ \hat{\mu}_x = \sum_{i=1}^{n} x_i / n \right\} \qquad (6)$$

where, $\hat{\mu}_x$ is the maximum likelihood estimate of μ_x. Similarly the solution of Equation (4) gives

$$\left\{ \hat{\sigma}_x = \sqrt{\sum_{i=1}^{n} (x_i - \mu_x)^2 / n} \right\} \qquad (7)$$

where, $\hat{\sigma}_x$ is the maximum likelihood estimate of σ_x.

b) <u>Log-Normal Distribution</u>

If the random variable x is log-normally distributed then $y = \log_e X$ is normally distributed. Let $y = N(\mu_y, \sigma_y^2)$, then according to Equation (4) in Appendix D-1, the probability density function of x is

$$\left\{ g(x) = \frac{1}{x \cdot \sigma_y \sqrt{2\pi}} e^{-(\log_e X - \mu_y)^2 / 2\sigma_y^2} \right\} \quad 0 < x \leq \infty \qquad (8)$$

The above function is completely specified if the parameters μ_y and σ_y are estimated. The likelihood function in this case may be computed as,

$$\left\{ L(x, \mu_y, \sigma_y) = \left(\frac{1}{\sigma_y\sqrt{2\pi}}\right)^n \cdot \frac{1}{\prod_{i=1}^{n} x_i} e^{-\frac{1}{2\sigma_y^2}\left(\sum_{i=1}^{n}(\log_e x_i - \mu_y)^2\right)} \right\} \quad (9)$$

Setting the derivatives of $\log_e(L)$ with respect to the parameters μ_y and σ_y, respectively, equal to zero and solving gives the maximum likelihood estimates of μ_y and σ_y:

$$\hat{\mu}_y = \sum_{i=1}^{n} \log_e x_i / n$$

$$\hat{\sigma}_y = \sqrt{\left(\sum_{i=1}^{n} \log_e x_i - \mu_y\right)^2 / n} \quad (12)$$

c) Gamma Distribution

The procedure to obtain the maximum likelihood estimates of the shape parameter, α, and the scale parameter, β, of a gamma distribution is detailed in a study by Markovic (1965). If the random variable, x, follows the gamma distribution, the probability density function can be represented by:

$$\left\{ f(x, \alpha, \beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}; \quad 0 \leq x \leq \infty \right\} \quad (13)$$

where $\Gamma(\alpha)$ is the gamma function.

The maximum likelihood function is

$$\left\{ L(x,\ \alpha,\ \beta) = \left(\frac{1}{\beta^{\alpha}\Gamma(\alpha)}\right)^{n} \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} e^{-\sum_{i=1}^{n} x_i/\beta} \right\} \tag{14}$$

and the natural log of the above equation gives

$$\log_e(L) = -n\alpha(\log_e \beta) - n \log_e(\Gamma(\alpha))$$

$$+\alpha \sum_{i=1}^{n} \log_e x_i - \sum_{i=1}^{n} \log_e(x_i) - \sum_{i=1}^{n} x_i/\beta \tag{15}$$

The parameters, α and β, can be estimated using partial derivatives of Equation (15) with respect to these parameters. Thus,

$$\frac{\partial}{\partial\beta}(\log_e(L)) = \frac{-n\hat{\alpha}}{\beta} + \frac{\sum_{i=1}^{n} x_i}{\hat{\beta}^2} = 0$$

or $$\hat{\beta} = \sum_{i=1}^{n} x_i/n\alpha = \hat{\mu}_X/\hat{\alpha} \tag{16}$$

Estimation of the shape parameter, α, by setting $\frac{\partial}{\partial\alpha}(\log_e(L)) = 0$ involves the digamma function $\frac{\partial}{\partial\alpha} \log_e(\Gamma(\alpha))$. Using an approximation of this function as illustrated by Markovic (1965), the expression for the maximum likelihood estimate of α is shown to be:

$$\hat{\alpha} = \frac{1 + \sqrt{1 + 4/3\left[\log_e \hat{\mu}_x - \frac{1}{n}\sum_{i=1}^{n}\log_e x_i\right]}}{4\left[\log_e \mu_x - \frac{1}{n}\sum_{i=1}^{n}\log_e x_i\right]} - \Delta\hat{\alpha} \qquad (17)$$

where, $\Delta\hat{\alpha}$ is a correction factor (Markovic, 1965).

Appendix D-3: Procedure for the Kolmogorov-Smirnov Test of Goodness of Fit.

The steps outlined below for each probability distribution describe the necessary procedure for performing the Kolmogorov-Smirnov goodness of fit test.

a) Normal Distribution

1. Estimate the mean and variance by the maximum likelihood method discussed in Appendix D-2;
2. Using the parameter estimates obtained in Step 1, plot the theoretical probability distribution function (cumulative probability distribution) on normal probability paper.
3. Rank the observed data in order of decreasing values, and compute the observed probability distribution function using the formula

$$p_j = \left(m_j/(n + 1)\right)$$

where,

p_j = non-exceedence probability of observation j

m_j = rank of observation j

n = sample size

4. Plot the observed probability distribution function and measure the maximum departure of these points from the theoretical distribution along the probability scale. This defines the Kolmogorov-Smirnov statistic, D.

5. Compare the statistic D with the critical values corresponding to the sample size, the desired level of significance and the probability distribution under consideration as given in Table 3-5, Chapter 3. If the value of D is less than the critical value, accept the hypothesis that the data is drawn from a population defined by the theoretical probability distribution.

b) Log-Normal Distribution

The procedure in this case is similar to the normal distribution since the goodness of fit test is applied to the transformed random variable $y = \log_e x$, where x is log-normal and y is normal.

c) Gamma Distribution (Markovic, 1965)

1. Transform the observed data into modular coefficients by dividing each observation by the mean.
2. Compute the parameters $\hat{\alpha}$ and $\hat{\beta}$ using the maximum likelihood procedure presented in Appendix D-2.
3. Select class intervals on the probability range 0 to 1, as shown in Table D-1.
4. Using Table D-1 and Figure D-1, obtain the values of u_j corresponding to each of the class intervals, j.
5. From the values u_j, the modular coefficients of the random variable may be computed by the equation:

$$\left[K_j = u_j/\sqrt{\hat{\alpha}} \right] \tag{1}$$

6. Plot the theoretical distribution function using the modular coefficients K_j computed in Step 5 and the probability class intervals, j.

TABLE D-1

INCOMPLETE GAMMA FUNCTION
FOR COMPUTATION OF CLASS INTERVAL LIMIT VALUES

Class interval, j		1	2	3	4	5	6
$I(u,p) = \frac{\Gamma_u(p+1)}{\Gamma_\infty(p+1)}$		0.14286	0.28571	0.42857	0.57143	0.71429	0.85714
$p = \hat{\alpha} - 1$	$\hat{\alpha}$	u_1	u_2	u_3	u_4	u_5	u_6
-0.8	0.2	0.007	0.015	0.036	0.092	0.303	0.932
-0.6	0.4	0.021	0.060	0.147	0.335	0.675	1.381
-0.4	0.6	0.048	0.140	0.299	0.540	0.919	1.630
-0.2	0.8	0.094	0.240	0.434	0.708	1.103	1.806
0.0	1.0	0.153	0.338	0.559	0.850	1.254	1.947
0.5	1.5	0.313	0.557	0.819	1.131	1.546	2.218
1.0	2.0	0.468	0.748	1.033	1.357	1.774	2.430
1.5	2.5	0.614	0.919	1.217	1.549	1.967	2.610
2	3	0.749	1.074	1.382	1.786	2.136	2.770
3	4	1.000	1.349	1.670	2.013	2.429	3.049
4	5	1.224	1.591	1.921	2.267	2.682	3.291
5	6	1.429	1.810	2.145	2.494	2.907	3.508
6	7	1.620	2.010	2.350	2.700	3.112	3.707
7	8	1.799	2.196	2.540	2.891	3.302	3.891
8	9	1.966	2.370	2.717	3.070	3.480	4.065
9	10	2.126	2.535	2.884	3.238	3.647	4.228
10	11	2.278	2.692	3.043	3.397	3.805	4.383
11	12	2.420	2.838	3.191	3.568	3.953	4.528
12	13	2.563	2.985	3.339	3.694	4.101	4.674
13	14	2.696	3.120	3.476	3.831	4.238	4.808
14	15	2.828	3.255	3.612	3.968	4.374	4.942
15	16	2.952	3.382	3.740	4.096	4.502	5.067
16	17	3.076	3.508	3.867	4.223	4.629	5.192
17	18	3.194	3.627	3.987	4.344	4.748	5.310
18	19	3.311	3.746	4.107	4.464	4.868	5.429
19	20	3.422	3.859	4.220	4.578	4.981	5.541
20	21	3.532	3.972	4.334	4.691	5.094	5.653
21	22	3.638	4.080	4.442	4.832	5.202	5.760
22	23	3.744	4.187	4.550	4.974	5.310	5.867
23	24	3.846	4.290	4.654	5.044	5.414	5.969
24	25	3.947	4.393	4.757	5.114	5.517	6.071
25	26	4.044	4.492	4.856	5.214	5.616	6.169
26	27	4.142	4.590	4.955	5.313	5.715	6.267
27	28	4.236	4.685	5.051	5.408	5.810	6.362
28	29	4.331	4.780	5.147	5.504	5.906	6.457
29	30	4.422	4.872	5.239	5.596	5.998	6.548
30	31	4.514	4.964	5.331	5.689	6.090	6.639
31	32	4.602	5.053	5.420	5.778	6.180	6.978
32	33	4.690	5.142	5.510	5.868	6.269	6.816
33	34	4.775	5.228	5.596	5.954	6.356	6.904
34	35	4.860	5.315	5.683	6.041	6.442	6.991
35	36	4.944	5.398	5.767	6.125	6.566	7.073
36	37	5.027	5.482	5.851	6.209	6.689	7.155
37	38	5.108	5.564	5.932	6.291	6.731	7.236
38	39	5.189	5.645	6.014	6.373	6.773	7.316
39	40	5.268	5.744	6.094	6.452	6.872	7.396
40	41	5.346	5.843	6.174	6.532	6.932	7.475
41	42	5.423	5.901	6.252	6.610	7.010	7.552
42	43	5.501	5.959	6.329	6.688	7.087	7.629
43	44	5.576	6.035	6.405	6.764	7.163	7.704
44	45	5.650	6.111	6.481	6.840	7.239	7.780
45	46	5.724	6.184	6.555	6.914	7.313	7.854
46	47	5.799	6.258	6.629	6.988	7.387	7.927
47	48	5.860	6.331	6.702	7.061	7.460	8.000
48	49	5.941	6.404	6.775	7.134	7.532	8.072
49	50	6.012	6.474	6.846	7.205	7.603	8.142
50	51	6.083	6.545	6.917	7.276	7.674	8.213
55	56	6.306	6.791	7.181	7.558	7.976	8.541
60	61	6.583	7.089	7.496	7.889	8.325	8.924
65	66	6.854	7.380	7.804	8.214	8.667	9.600
70	71	7.124	7.672	8.112	8.538	9.010	9.648

Table D-1 (continued)

$p = \hat{\alpha} - 1$	$\hat{\alpha}$	u_1	u_2	u_3	u_4	u_5	u_6
80	81	7.605	8.190	8.660	9.114	9.618	10.300
90	91	8.080	8.701	9.201	9.684	10.218	10.943
100	101	8.507	9.161	9.687	10.195	10.758	11.521
110	111	8.922	9.607	10.159	10.692	11.283	12.083
120	121	9.319	10.035	10.611	11.168	11.785	12.620

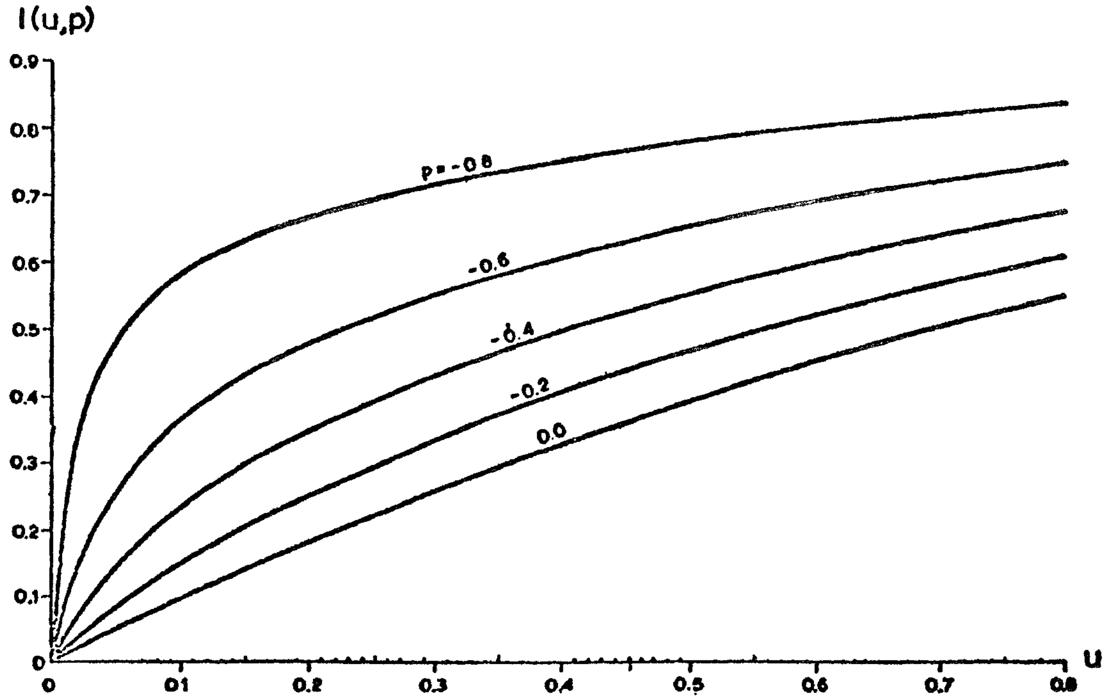


Figure D-1: Incomplete Gamma Function Used Only for $p \leq 0$ and $u \leq 0.800$

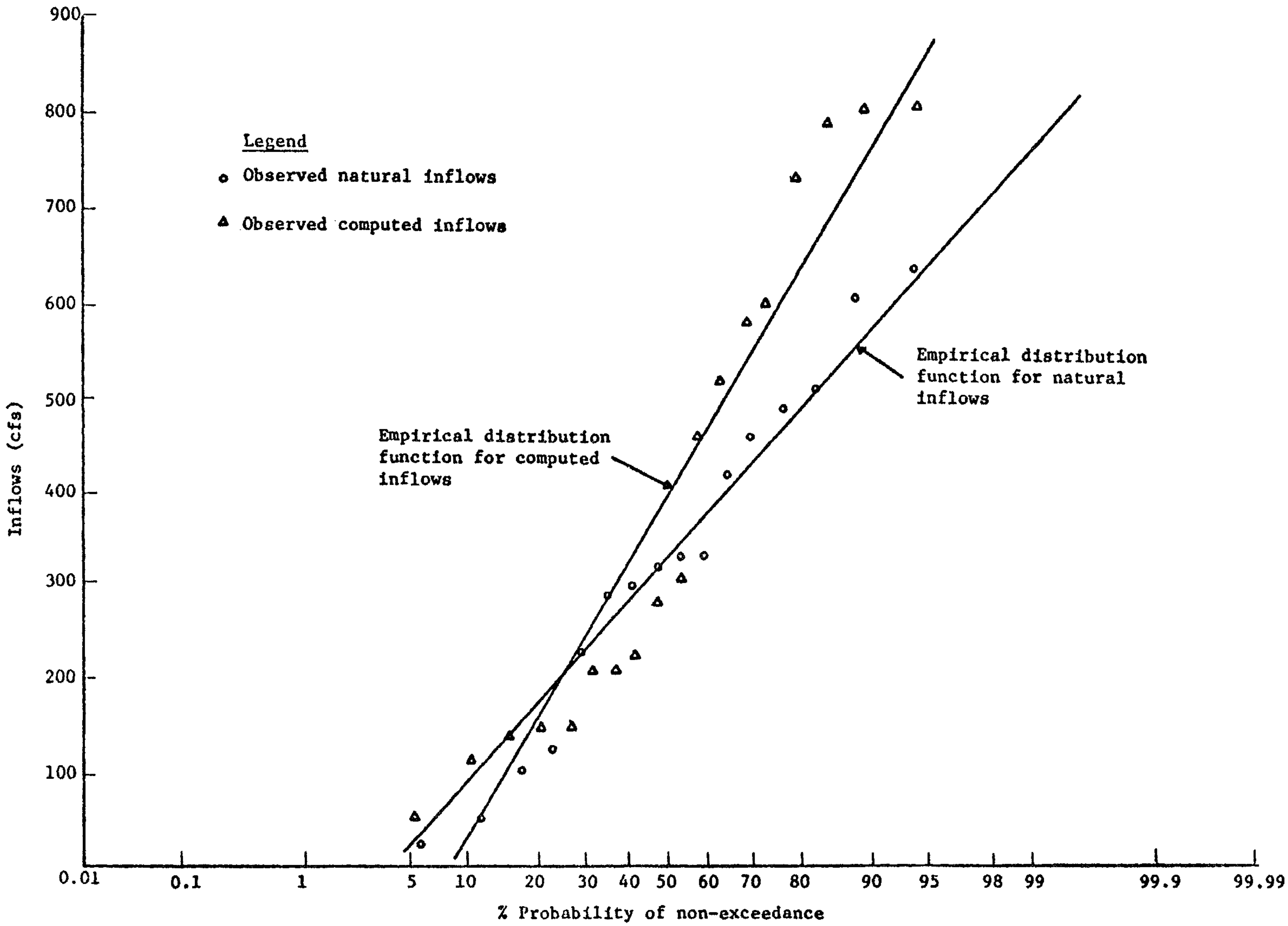


Figure D-2 (a): Frequency Distribution for Normal Inflows in April

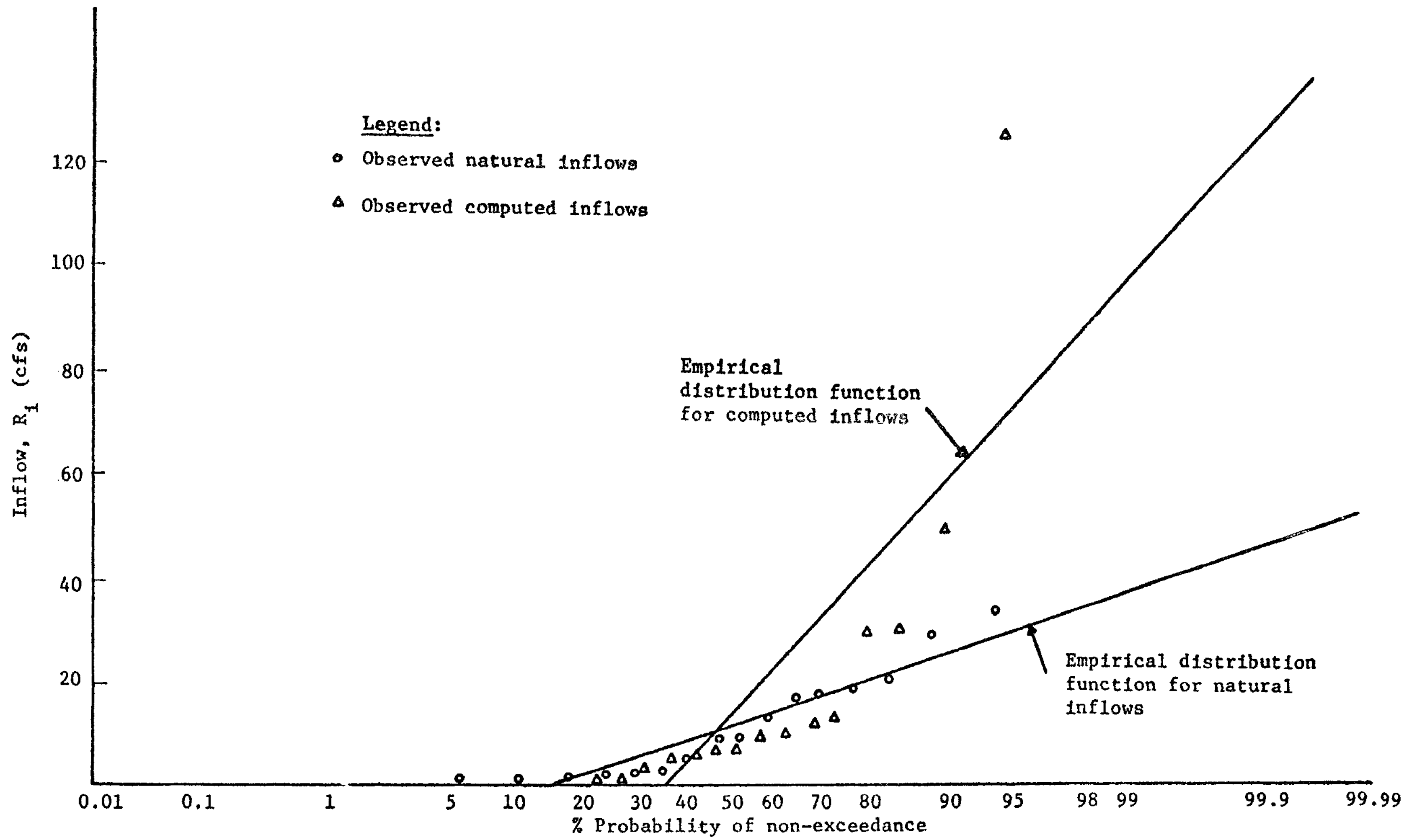


Figure D-2 (b): Frequency Distribution for Normal Inflows in October

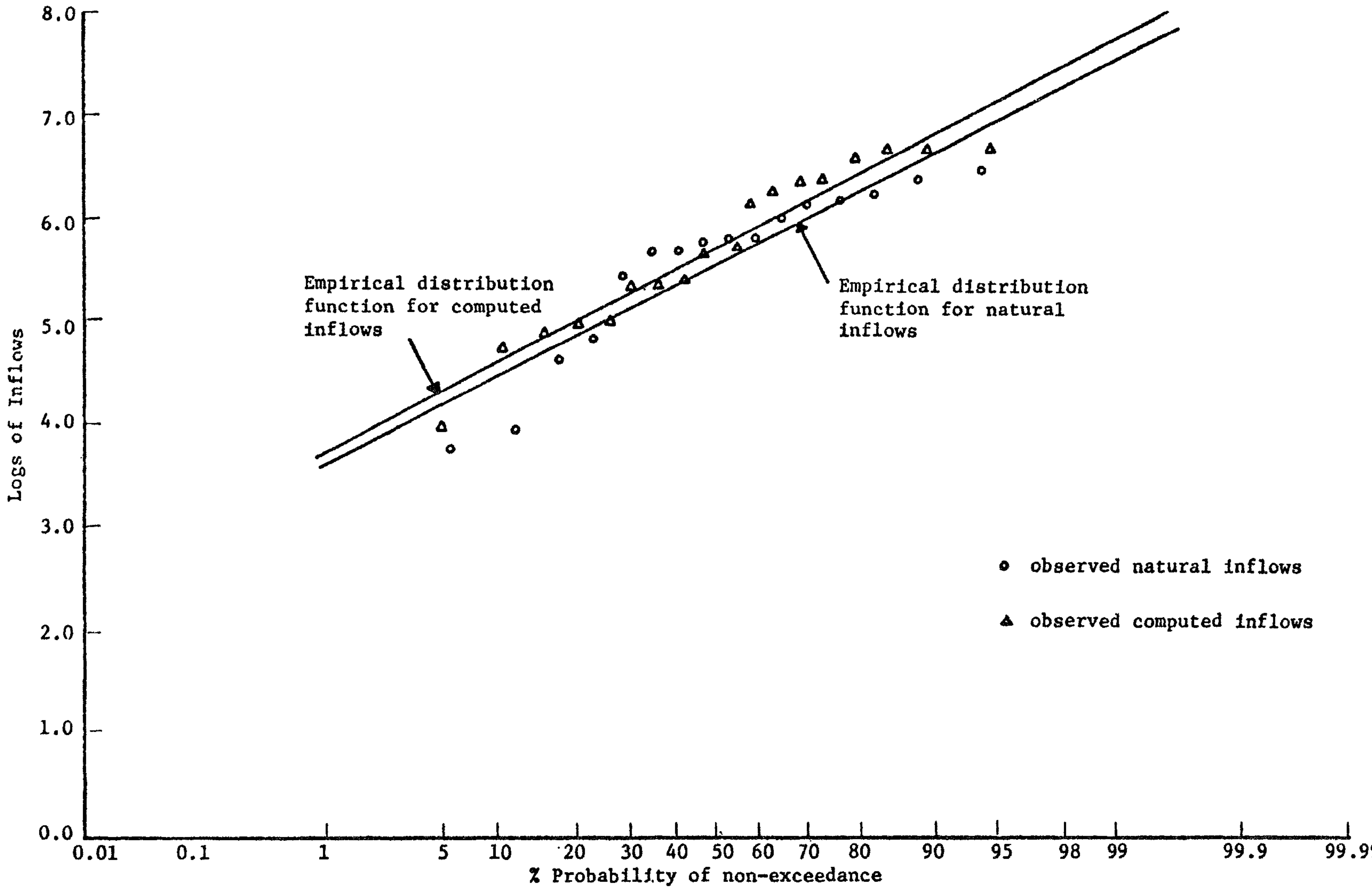


Figure D-3 (a): Frequency Distribution for Log-Normal Inflows in April

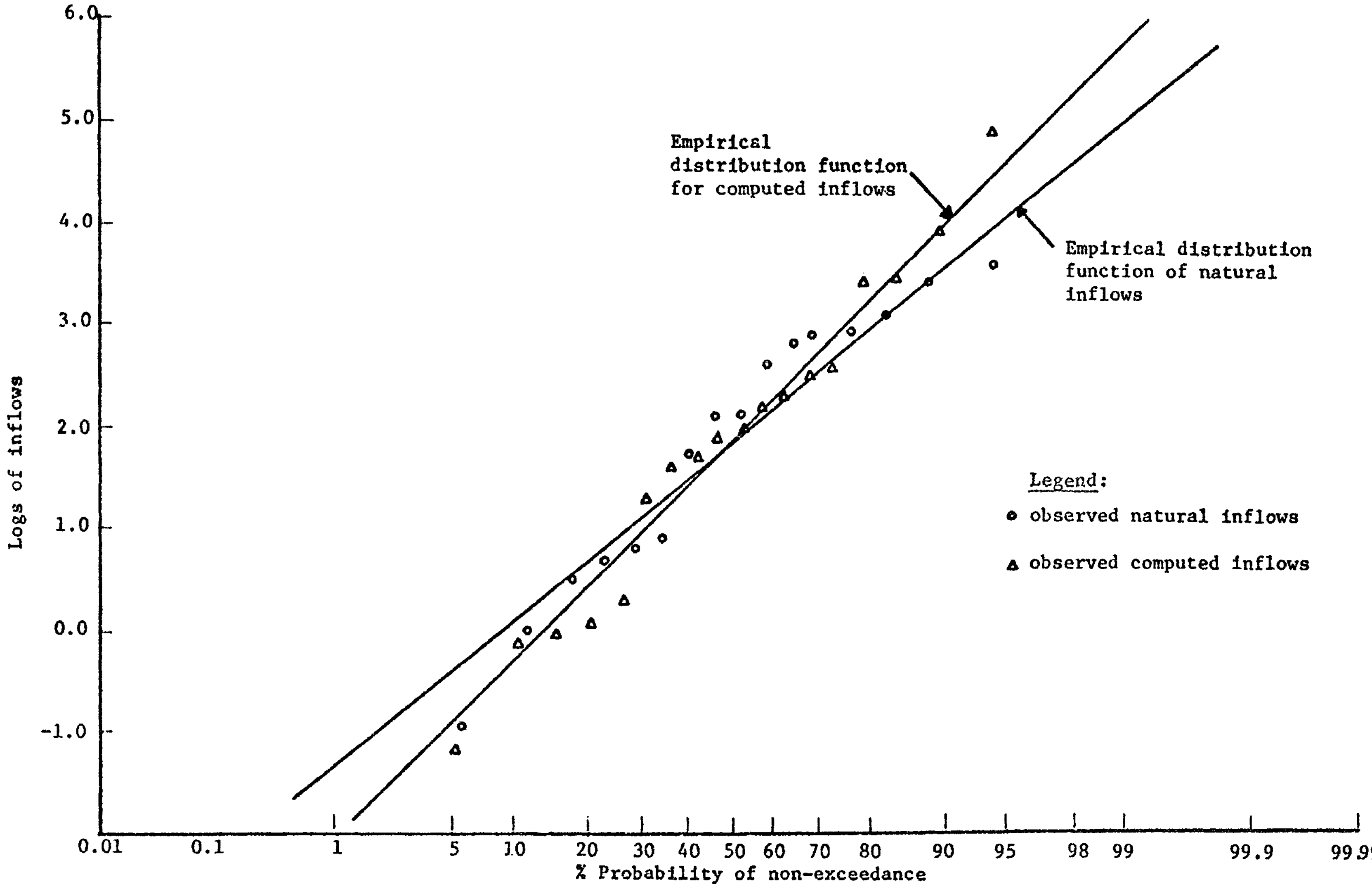


Figure D-3 (b): Frequency Distribution of Log-Normal Inflows in October

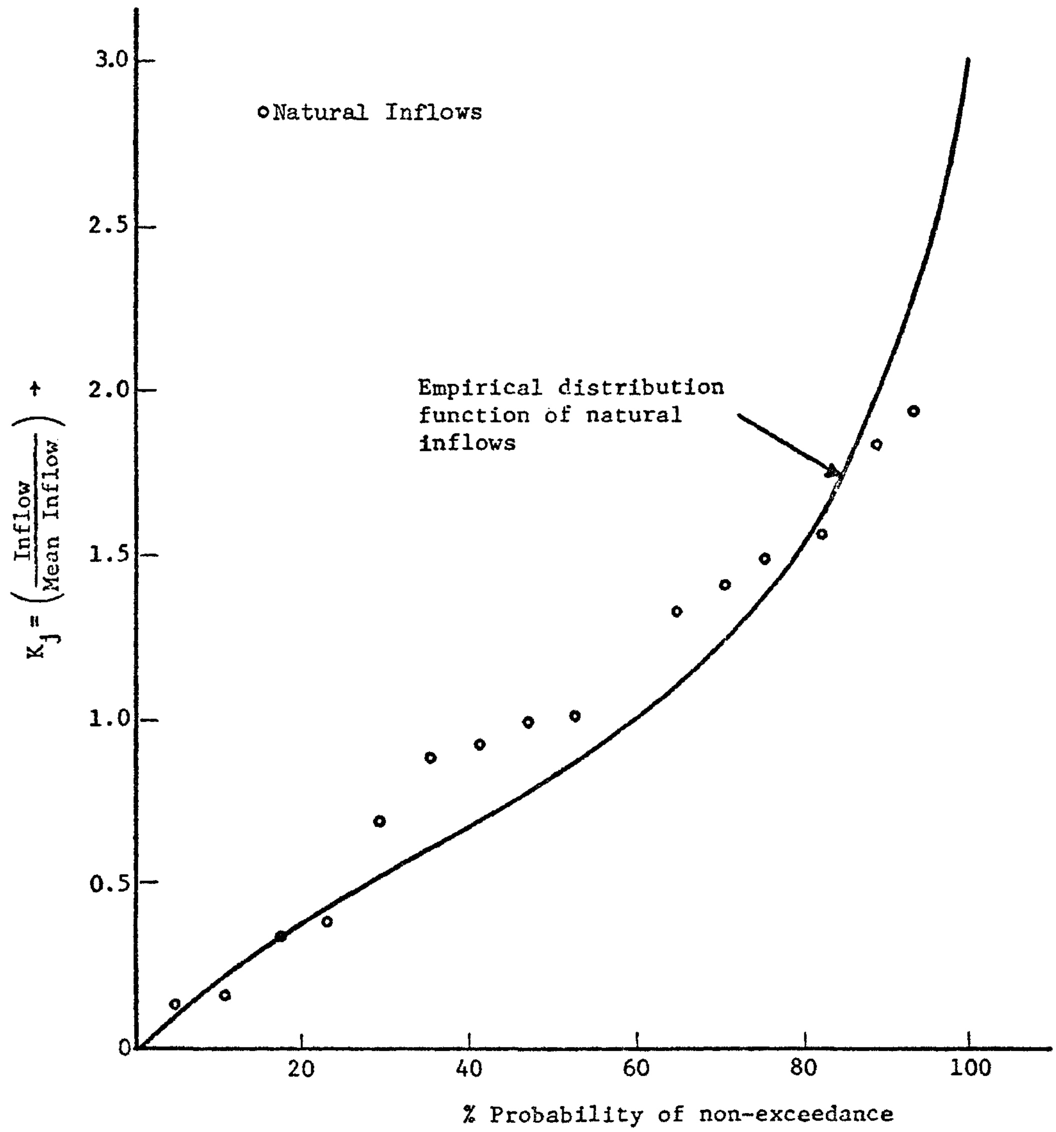


Figure D-4 (a): Frequency Distribution for Gamma Inflows in April

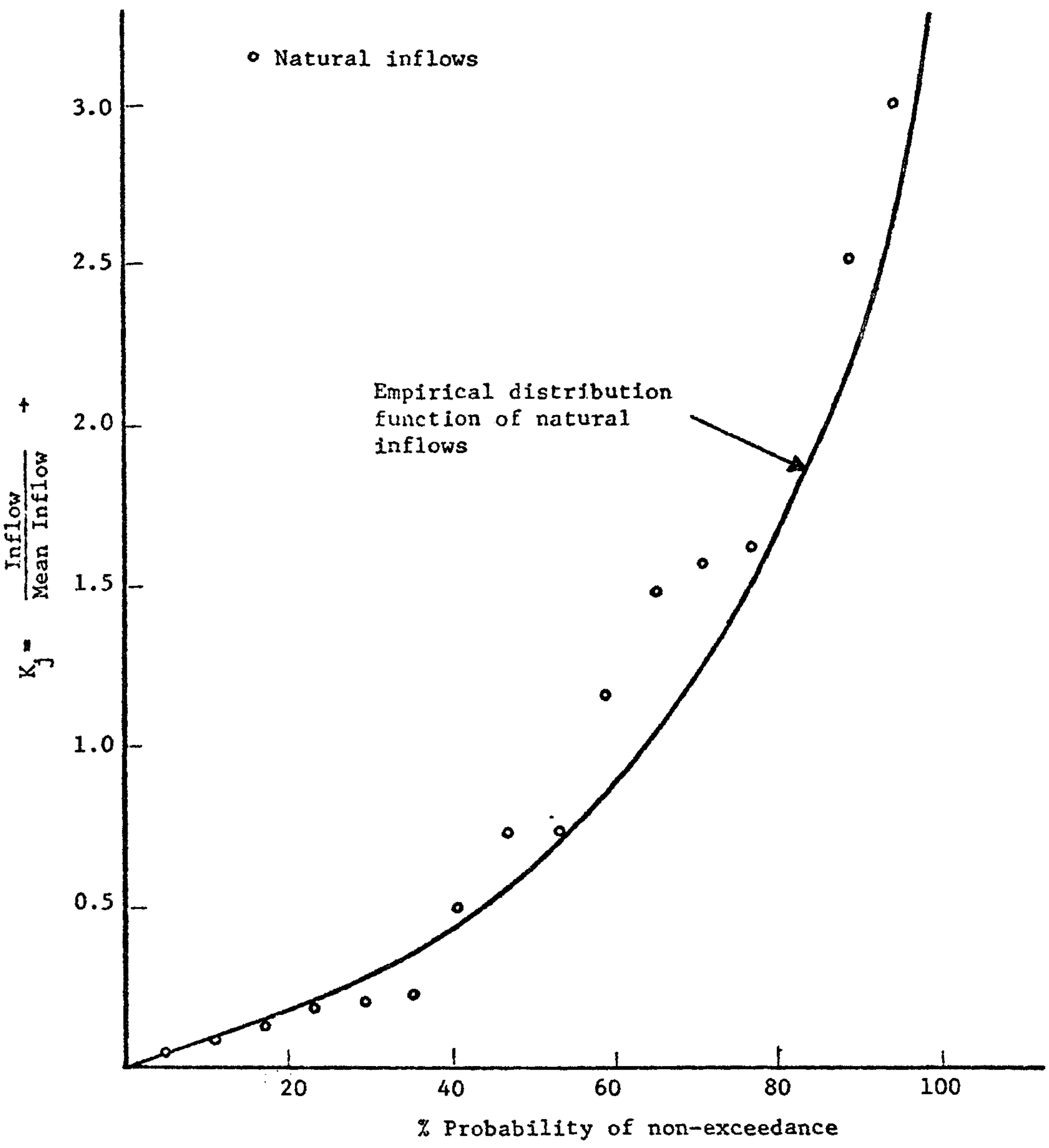


Figure D-4 (b): Frequency Distribution for Gamma Inflows in October

7. Follow the Kolmogorov-Smirnov test procedure outlined in Steps 3, 4 and 5 under the normal distribution case.

Figures D-2, D-3, and D-4 illustrate graphically the theoretical and observed probability distribution functions of natural inflows for the normal, log-normal and gamma cases, respectively. The results of the frequency analysis performed on the computed inflows (inflows after the construction of the reservoir) are also included for the normal and log-normal cases.

APPENDIX E

Chance-Constrained Model Operating Policies

Table E-1: Optimal Monthly Release Policies for Chance-Constrained Linear Programming Models (all units in thousand acre-feet)

a) Model LDRI

Month, i	$\alpha^* = 0.90$				$\alpha^* = 0.80$			
	$\alpha' = 0.98$		$\alpha' = 0.80$		$\alpha' = 0.98$		$\alpha' = 0.80$	
	b_i	$b_{i-1}-b_i$	b_i	$b_{i-1}-b_i$	b_i	$b_{i-1}-b_i$	b_i	$b_{i-1}-b_i$
January	1.9503	0.9799	1.7153	2.1703	0.2428	0.9787	-0.7177	2.1690
February	1.4304	0.5199	1.7701	-0.0548	-0.2784	0.5211	-0.6642	-0.0535
March	3.1223	-1.6919	6.6051	-4.8350	1.4136	-1.6919	4.1708	-4.8350
April	7.5453	-4.4230	15.6717	-9.0666	5.8366	-4.4230	13.2374	-9.0666
May	9.0983	-1.5530	19.8791	-4.2074	7.3896	-1.5530	17.4448	-4.2074
June	8.9185	0.1798	19.6813	0.1978	7.2098	0.1798	17.2470	0.1978
July	8.2337	0.6848	18.5821	1.0992	6.5250	0.6848	16.1478	1.0992
August	7.2433	0.9904	16.0997	2.4824	5.5346	0.9904	13.6654	2.4824
September	6.1574	1.0859	12.9995	3.1002	4.4487	1.0859	10.5652	3.1002
October	5.0624	1.095	9.8231	3.1764	3.3537	1.0950	7.3888	3.1764
November	3.9839	1.0785	6.7106	3.1125	2.2752	1.0785	4.2763	3.1125
December	2.9302	1.0537	3.8856	2.8250	1.2215	1.0537	1.4513	2.8250
Optimal Capacity C	95.0424		94.8074		63.4042		69.7882	

b) <u>Model LDR2</u>

January	1.2491	0.5199	1.6985	-0.0548	1.2491	0.5199	1.6985	-0.0548
February	2.9398	-1.6907	6.5295	-4.8337	2.9398	-1.6907	6.5295	-4.8337
March	7.3628	-4.4230	15.5961	-9.0666	7.3628	-4.4230	15.5961	-9.0666
April	8.9158	-1.5530	19.8035	-4.2074	8.9158	-1.5530	19.8035	-4.2074
May	8.7360	0.1798	19.6057	0.1978	8.7360	0.1798	19.6057	0.1978
June	8.0512	0.6848	18.5065	1.0992	8.0512	0.6848	18.5065	1.0992
July	7.0608	0.9904	16.0241	2.4824	7.0608	0.9904	16.0241	2.4824
August	5.9749	1.0859	12.9239	3.1002	5.9749	1.0859	12.9239	3.1002
September	4.8799	1.0950	9.7475	3.1764	4.8799	1.0950	9.7475	3.1764
October	3.8014	1.0785	6.6350	3.1125	3.8014	1.0785	6.6350	3.1125
November	2.7477	1.0537	3.8100	2.8250	2.7477	1.0537	3.8100	2.8250
December	1.7690	0.9787	1.6410	2.1690	1.7690	0.9787	1.6410	2.1690
Optimal Capacity C	34.6966		45.5843		34.6966		45.5843	

Chance-Constrained Linear Programming Using a Two-Sided Quadratic Loss Function: The chance-constrained linear programming formulations, discussed in Chapter 6, were solved with the objective of obtaining the minimum reservoir capacity to satisfy the imposed reliability constraints. The purpose of this section is to present results which show that, under a given optimal capacity C, the optimal release policies remain unaltered if a quadratic loss function is substituted as the objective function.

The quadratic loss function to be minimized is

$$\text{minimize } Z = E\left[\sum_{i=1}^{12} w_i(X_i - T_i)^2\right] \tag{E-1}$$

where E = the expected value operator;

w_i = a priority weight associated with month i;

T_i = desired target release in month i.

Expanding Equation E-1,

$$E\left[\sum_{i=1}^{12} w_i(X_i - T_i)^2\right] = \sum_{i=1}^{12} E\left[w_i(X_i^2 - 2X_i \cdot T_i + T_i^2\right]$$

$$= \left[\sum_{i=1}^{12} w_i E(X_i^2) - \sum_{i=1}^{12} 2 \cdot T_i \cdot E(X_i) + \sum_{i=1}^{12} T_i^2)\right]$$

and using the definition,

$$E(X_i^2) = Var(X_i) + \left(E(X_i)\right)^2$$

where $Var(X_i)$ = the variance of the release X_i,

the final form of the loss function may be expressed as:

$$\text{minimize } Z = \sum_{i=1}^{12} w_i Var(X_i) + \sum_{i=1}^{12} w_i \left(E(X_i) - T_i \right)^2 \qquad \text{(E-2)}$$

For the original linear release policies illustrated in Equations 6-1 and 6-4, the first term in Equation E-2 may be omitted, since the variance of the release depends on the random inflows whose variance cannot be controlled. Consequently, the loss function to be minimized reduces to:

$$\text{minimize } Z = \sum_{i=1}^{12} A_i[E(X_i)]^2 + B_i E(X_i) + C_i \qquad \text{(E-3)}$$

where $E(X_i) = E(R_{i-1}) + b_{i-1} - b_i$ for model LDR1

$E(X_i) = E(R_i) + b_{i-1} - b_i$ for model LDR2

$A_i = w_i$; $B_i = -2W_iT_i$; $C_i = T_i^2$

The nonlinear term in Equation E-3 is a function of the relative magnitude of the decision constants, as expressed by the difference, $b_{i-1} - b_i$. If the feasibility requirement of the chance constraints on minimum guaranteed flow (Equations 5-20 and 5-21) is satisfied as a strict equality, then values of $b_{i-1} - b_i$ are solely determined by these constraints. Consequently, the solution of the chance-constrained linear program under the quadratic loss function (Equation E-3) and the chance constraints presented in Chapter 6 would be identical to the solution under the capacity minimization problem. It must be emphasized that the optimal capacity obtained by solving the capacity minimization problem is used in the chance-constrained formulation using the quadratic loss function. Also, the reliabilities are maintained at the same level in both the problems.

APPENDIX F

Performance Characteristics of Chance-Constrained Models at Low Targets

Table F-1: Performance Characteristics of Chance-Constrained Models at a Target of 12 MGD (all units in thousand acre-feet)

	Model LDR1*		Model LDR2**	
Month	Original	Transformed	Original	Transformed
I. Average Monthly Releases				
January	15.427	15.581	22.424	22.423
February	19.567	21.073	19.114	19.114
March	20.086	19.970	19.982	19.982
April	19.449	20.264	18.033	18.035
May	17.363	17.380	11.468	11.468
June	11.862	11.778	12.646	12.646
July	11.951	11.885	6.795	6.795
August	6.772	5.909	3.963	3.963
September	3.922	3.242	2.341	2.341
October	2.341	1.861	1.757	1.757
November	1.759	1.703	5.308	5.308
December	6.191	6.112	12.863	12.862
II. Average Beginning Monthly Storages				
January	12.560	11.351	1.769	1.768
February	19.037	17.674	1.249	1.249
March	20.275	17.406	2.940	2.940
April	24.593	21.840	7.361	7.361
May	24.732	21.164	8.916	8.914
June	18.657	15.072	8.736	8.734
July	18.756	15.255	8.051	8.049
August	12.610	9.176	7.061	7.059
September	8.714	6.145	5.975	5.973
October	6.038	4.149	4.880	4.878
November	4.376	2.967	3.801	3.799
December	6.871	5.518	2.748	2.745

III. Average Monthly Shortages (Percent)

January	0.00	0.07	0.00	0.00
February	0.00	0.00	0.00	0.00
March	0.00	0.00	0.71	0.71
April	0.68	0.68	0.10	0.10
May	0.10	0.14	0.17	0.17
June	0.17	0.27	0.07	0.07
July	0.07	1.18	0.00	0.00
August	0.00	8.24	0.51	0.51
September	0.07	16.66	0.03	0.03
October	1.59	29.63	0.27	0.27
November	0.00	7.94	0.00	0.00
December	0.24	1.01	0.00	0.00

IV. Average Monthly Loss

January	666.587	691.559	1024.741	1024.710
February	525.910	695.799	459.850	459.850
March	527.355	529.931	554.125	554.124
April	487.889	571.952	424.750	424.778
May	348.402	377.784	191.600	191.600
June	234.290	241.387	275.762	275.762
July	197.092	228.750	66.812	66.812
August	62.831	52.840	31.965	31.965
September	28.507	23.049	5.534	5.534
October	5.442	4.236	0.772	0.772
November	0.835	0.811	34.777	34.776
December	112.287	107.390	324.818	324.805

V. Negative Predicted Releases (Percent)

January	0.00	0.00	0.00	0.00
February	0.00	0.00	0.00	0.00
March	0.00	0.00	0.24	0.24
April	0.24	0.24	0.00	0.00
May	0.00	0.00	0.00	0.00
June	0.00	0.00	0.00	0.00
July	0.00	0.00	0.00	0.00
August	0.00	0.00	0.00	0.00
September	0.00	0.00	0.00	0.00
October	0.00	0.00	0.00	0.00
November	0.00	0.00	0.00	0.00
December	0.00	0.00	0.00	0.00

VI. Month End Maximum Storage Violations (Percent)

January	11.76	10.47	0.00	0.00
February	5.37	3.45	0.00	0.00
March	12.03	9.22	63.92	0.00
April	8.51	5.71	0.00	24.56
May	2.64	2.13	0.00	0.00
June	4.53	2.77	0.00	0.00
July	0.54	0.17	0.00	0.00
August	0.34	0.20	0.00	0.00
September	0.00	0.00	0.00	0.00
October	0.00	0.00	0.00	0.00
November	0.07	0.03	0.00	0.00
December	3.48	3.21	0.00	0.00

VI. Month End Minimum Storage Violations (Percent)

January	0.91	3.38	0.00	37.47
February	0.00	4.97	0.00	0.00
March	0.00	3.28	0.00	0.00
April	0.00	4.29	0.00	0.00
May	0.00	9.53	0.00	0.00
June	0.00	6.89	0.00	0.00
July	0.00	15.51	0.00	0.00
August	0.00	21.89	0.00	0.00
September	0.00	26.08	0.00	0.00
October	0.00	43.85	0.00	0.00
November	0.00	13.82	0.00	0.07
December	0.51	5.47	0.00	0.14

*Optimal Capacity = 67.6025 th.ac.ft.
SMAX = 41.8217 th.ac.ft.
SMIN = 2.4337 th.ac.ft.
$\alpha^* = 0.83$; $\alpha' = 0.98$

**Optimal Capacity = 34.6965 th.ac.ft.
SMAX = 8.9157 th.ac.ft.
SMIN = 1.2491 th.ac.ft.
$\alpha^* = 0.83$; $\alpha' = 0.98$

APPENDIX G

Scatter Plots of Optimal Releases Obtained by Dynamic Programming

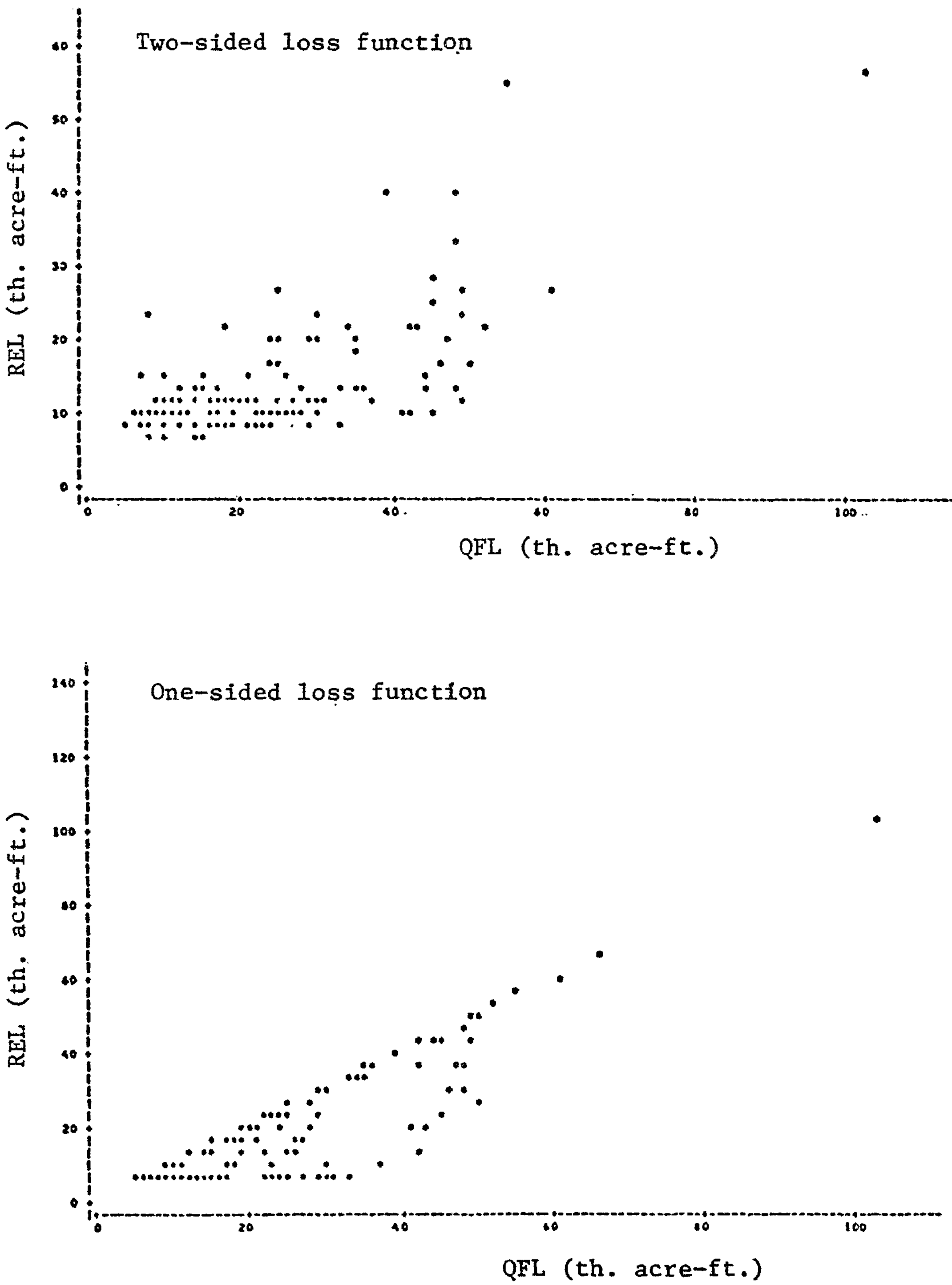
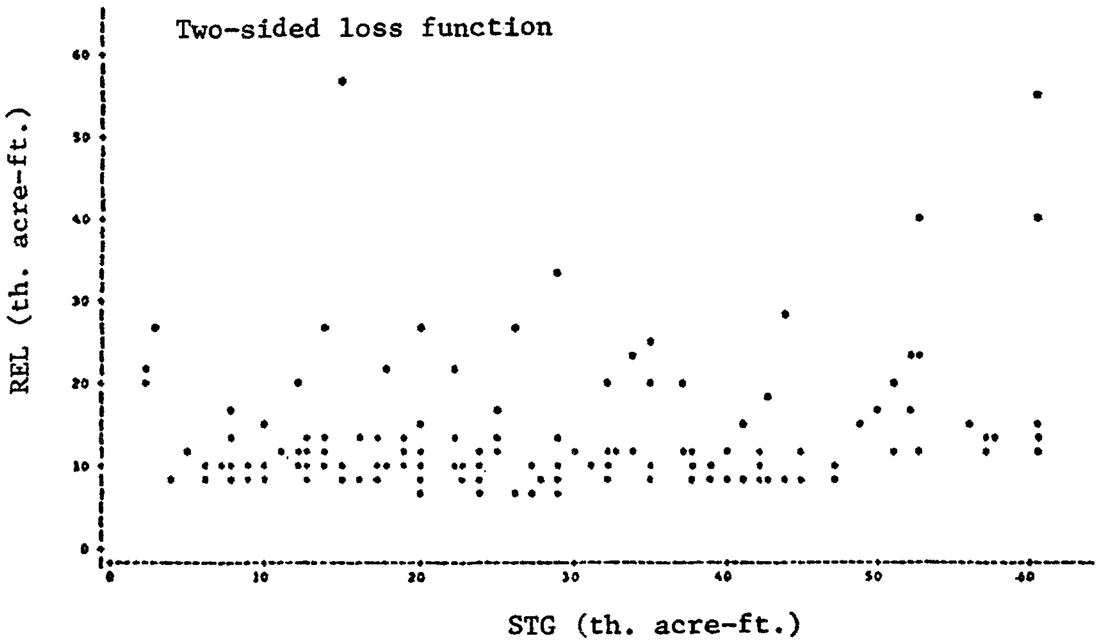


Figure G-1: Scatter Plot of Optimal Releases for Different Inflows in March (Target = 75 MGD).



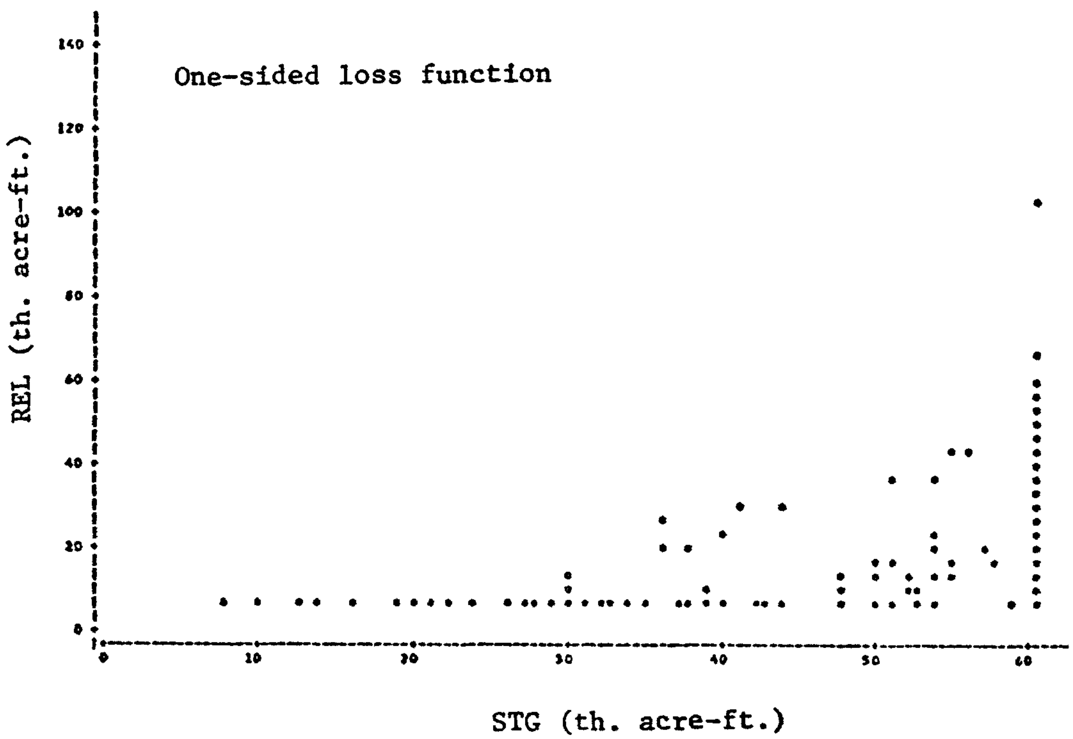
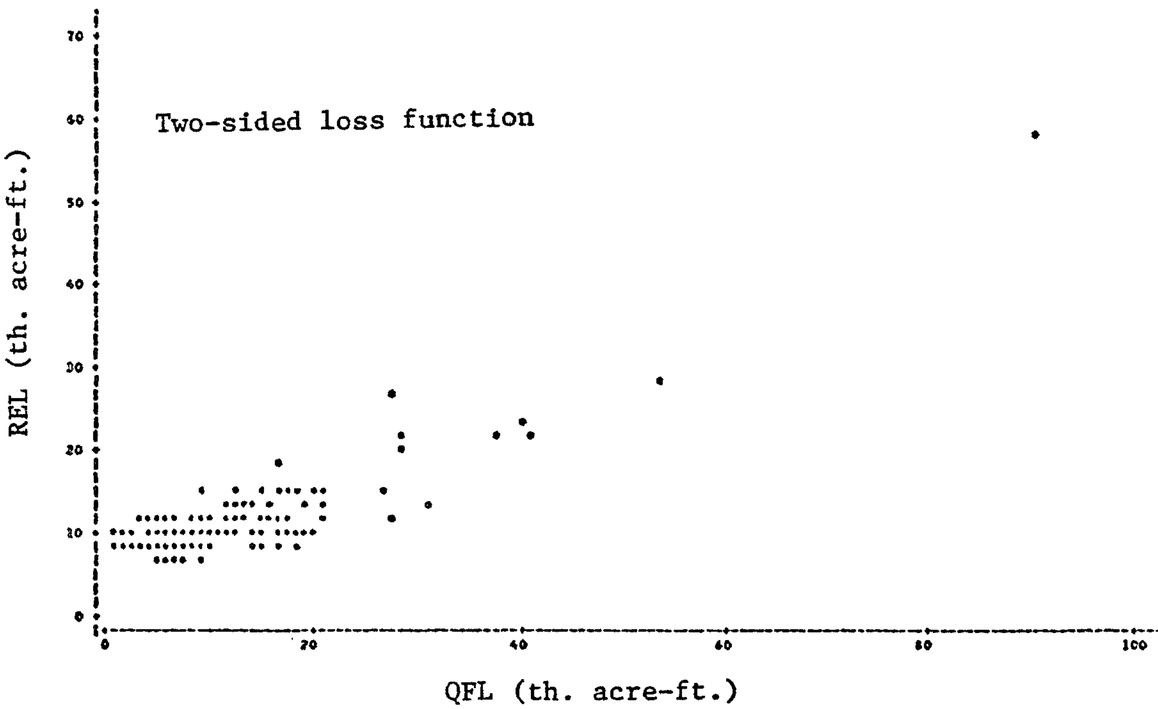


Figure G-2: Scatter Plots of Optimal Releases at Different Levels of Storage in March (Target = 75 MGD)



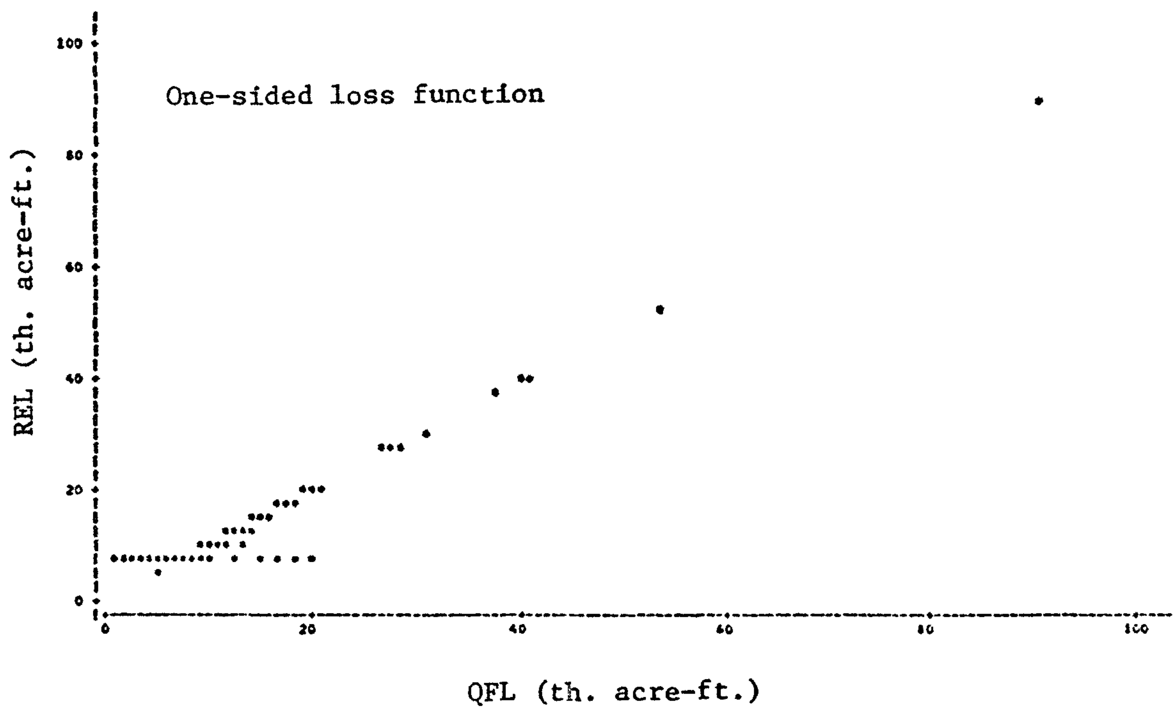
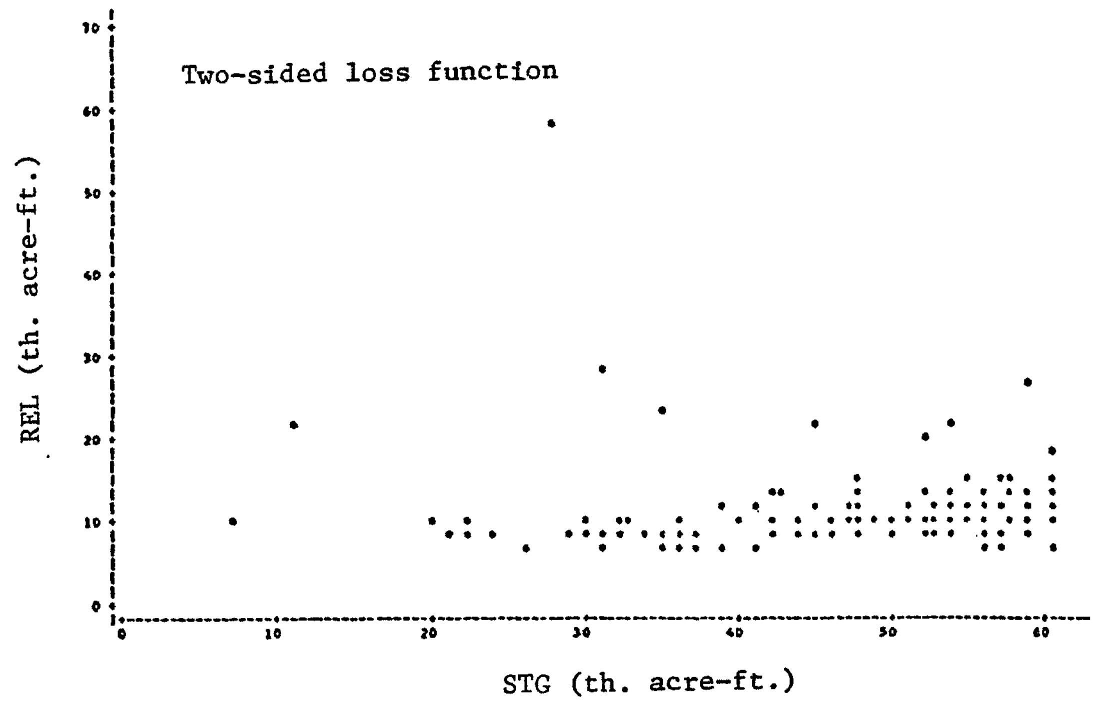


Figure G-3: Scatter Plot of Optimal Releases for Different Inflows in June (Target = 75 MGD)



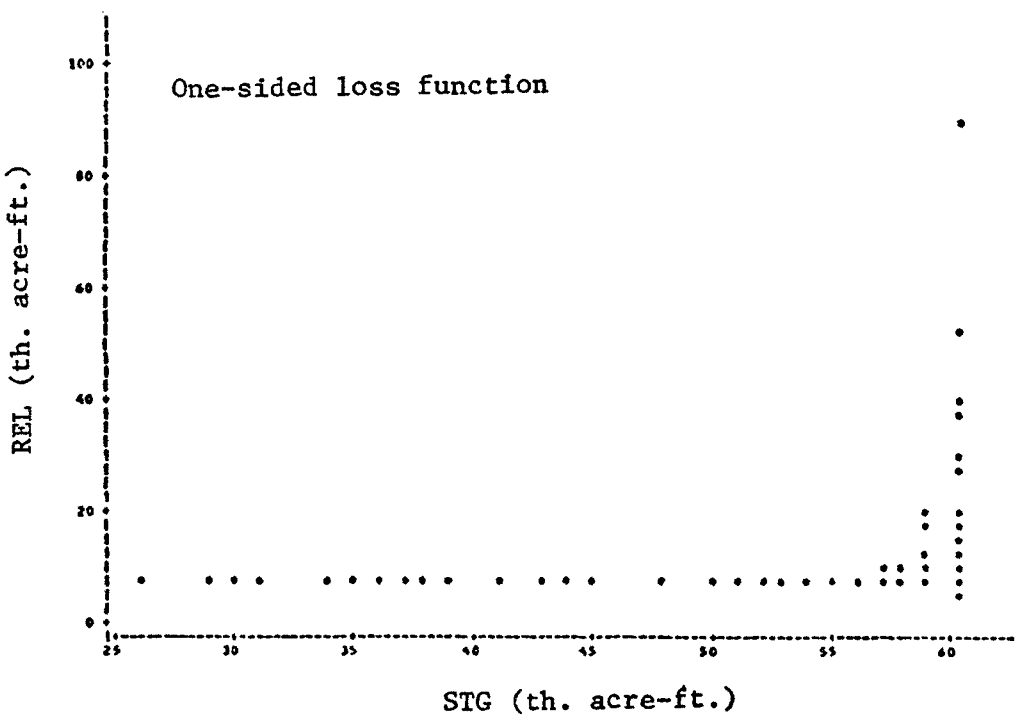
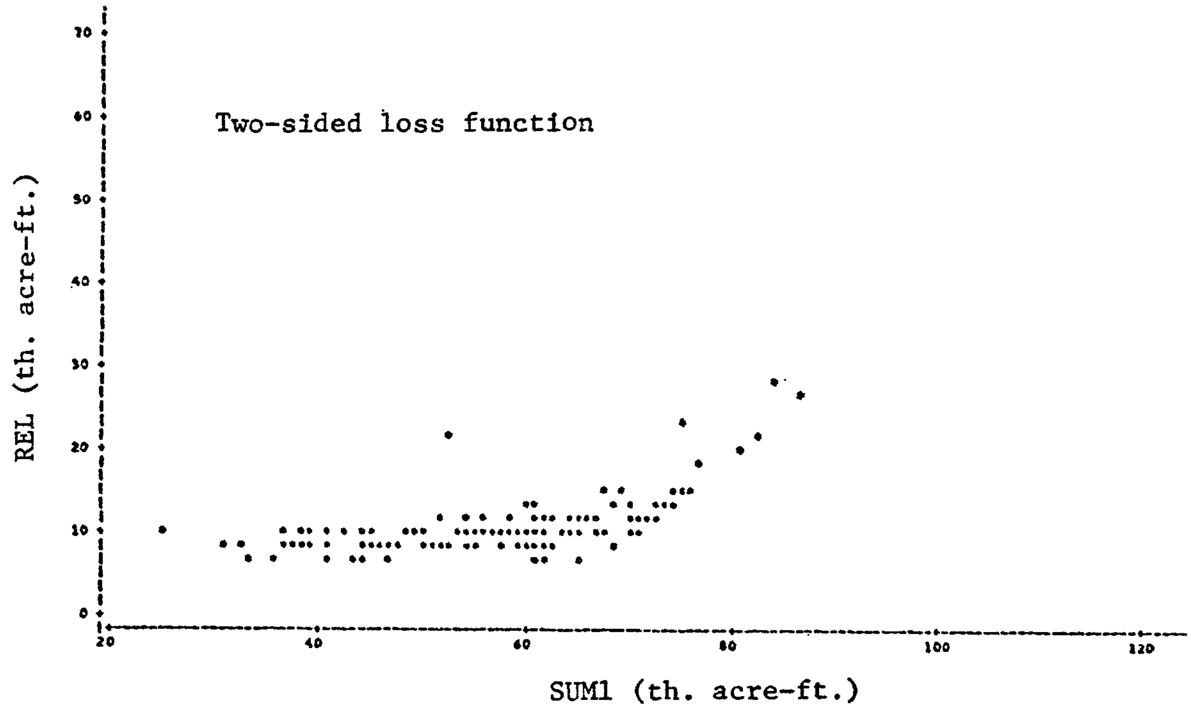


Figure G-4: Scatter Plot of Optimal Releases at Different Levels of Storage in June (Target = 75 MGD)



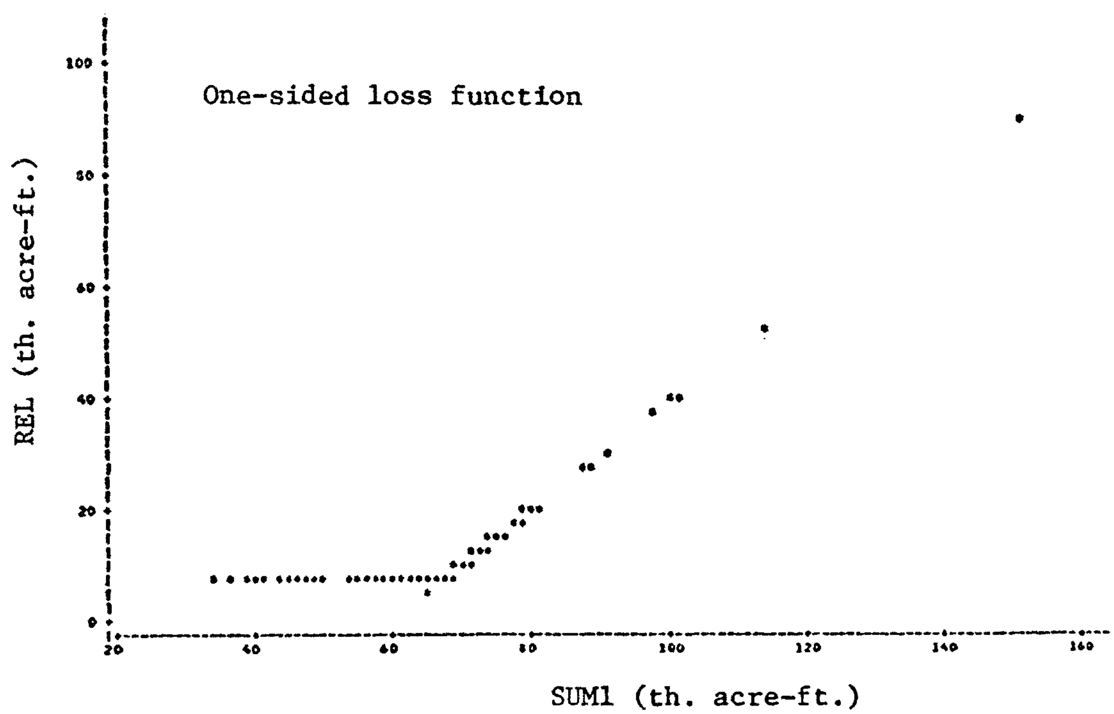
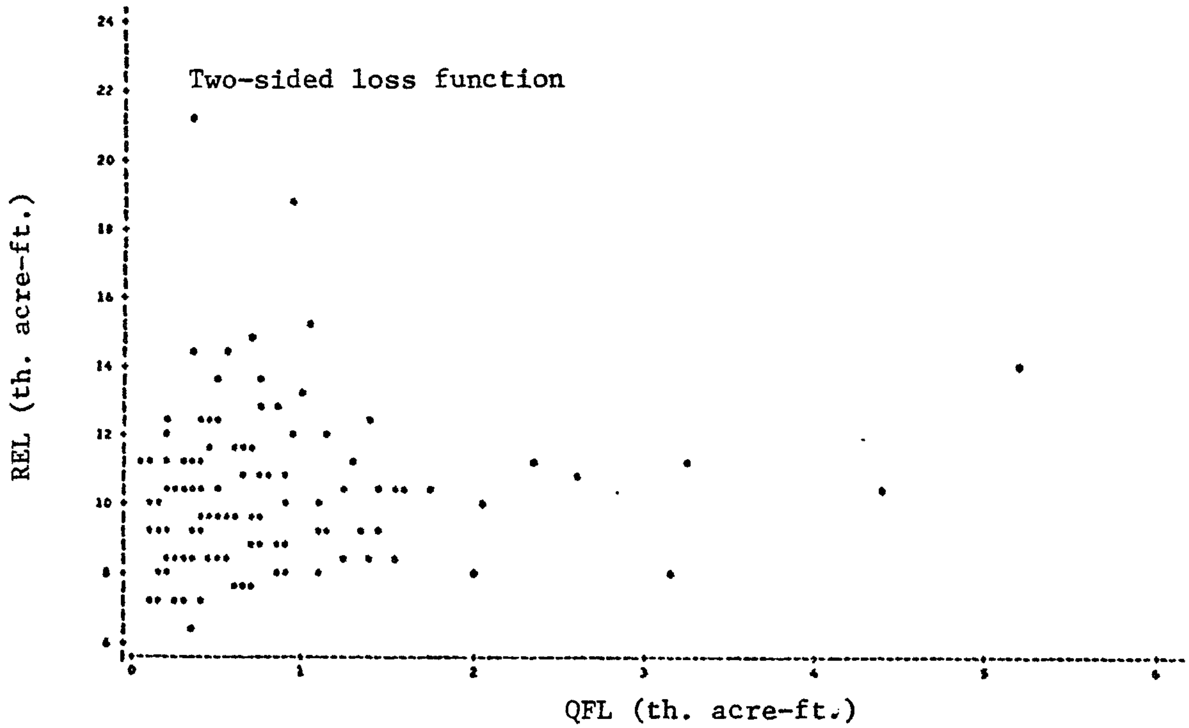


Figure G-5: Scatter Plot of Optimal Releases at Different Levels of (Inflow + Storage) in June (Target = 75 MGD)



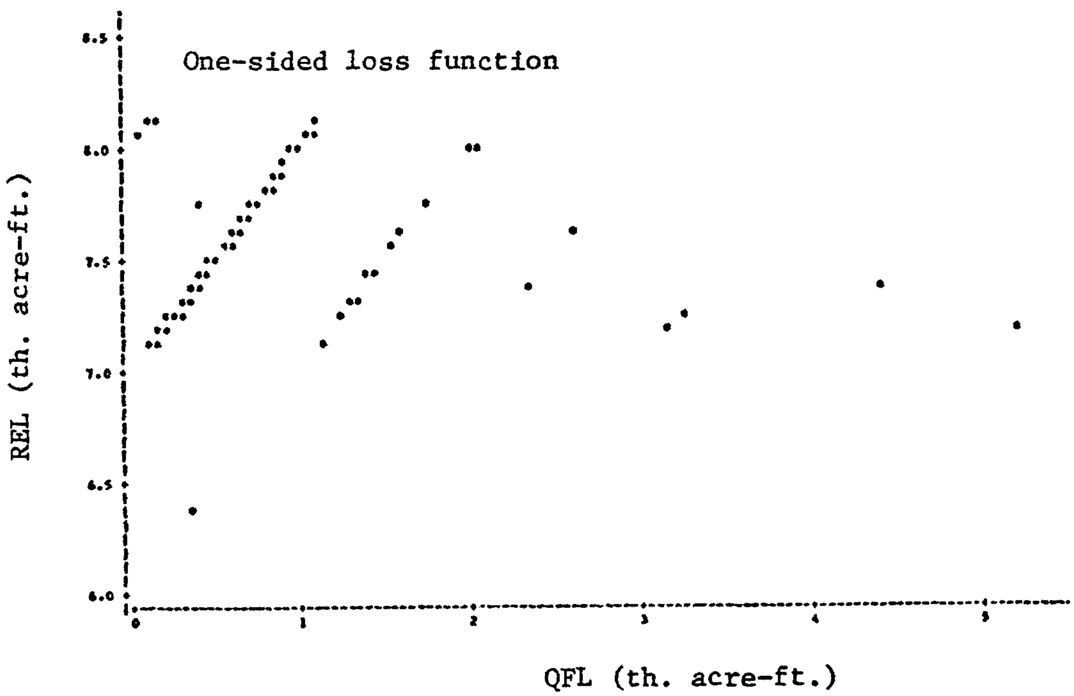
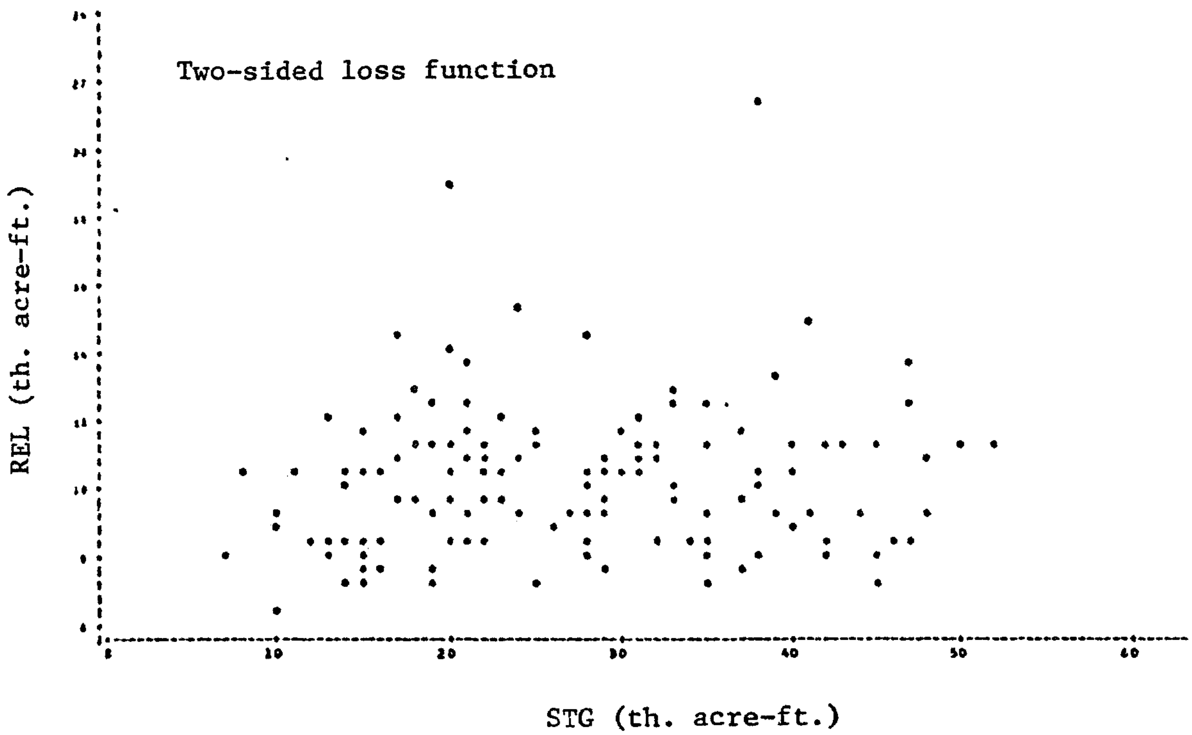


Figure G-6: Scatter Plot of Optimal Releases at Different Inflows in October (Target = 75 MGD)



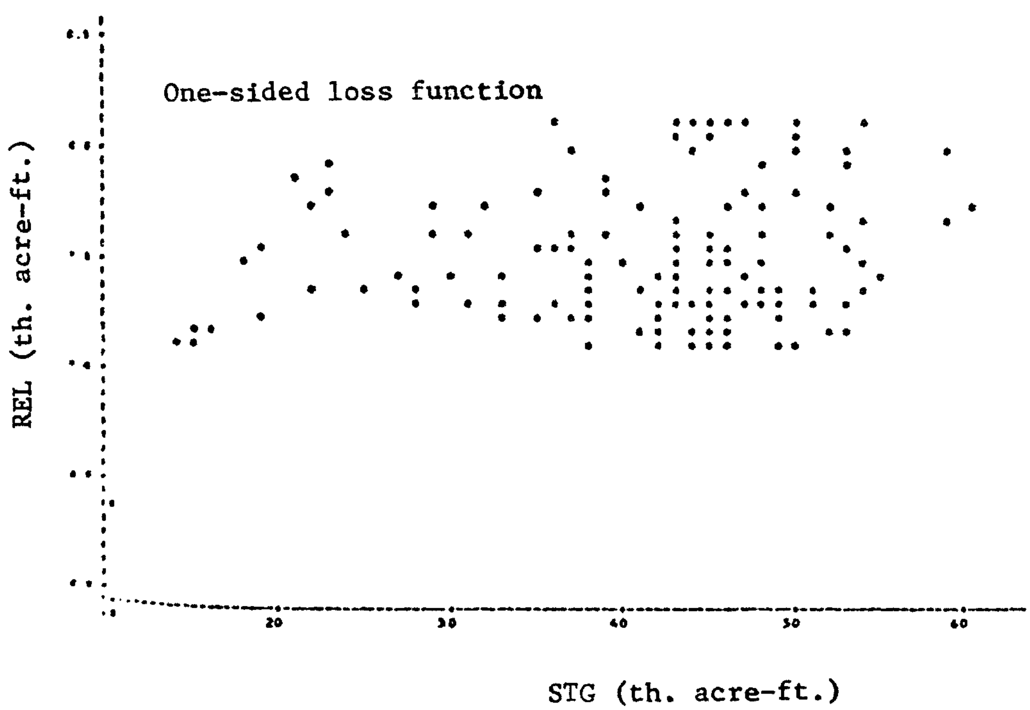
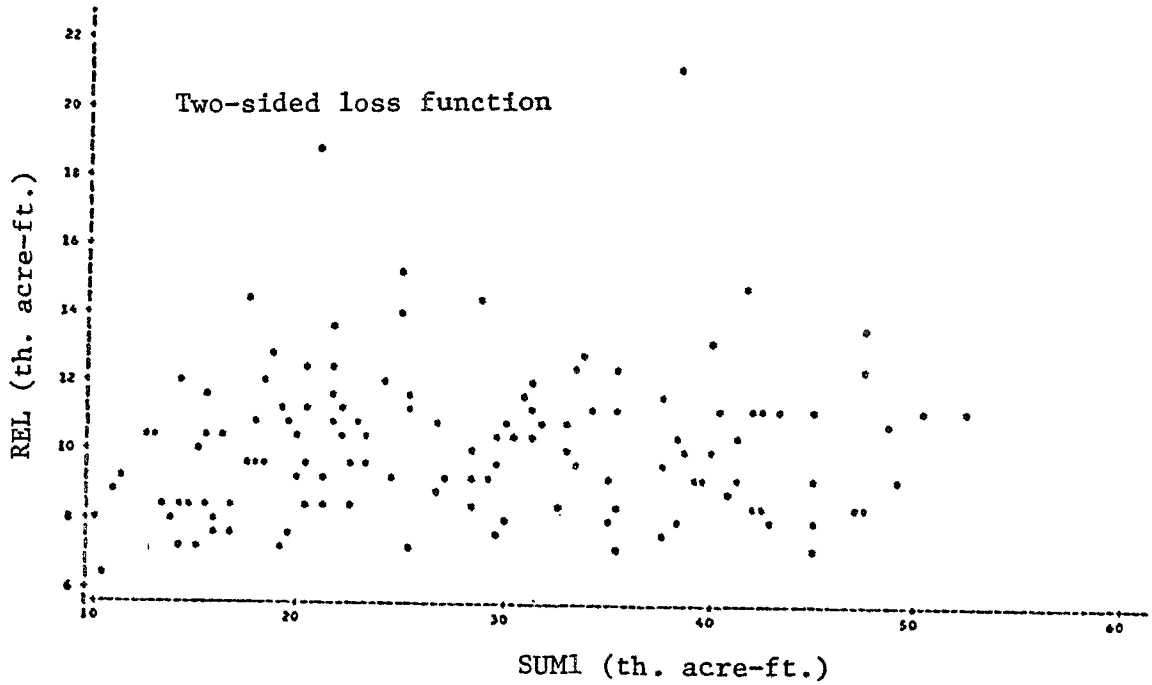


Figure G-7: Scatter Plot of Optimal Releases at Different Storage Levels in October (Target = 75 MGD)



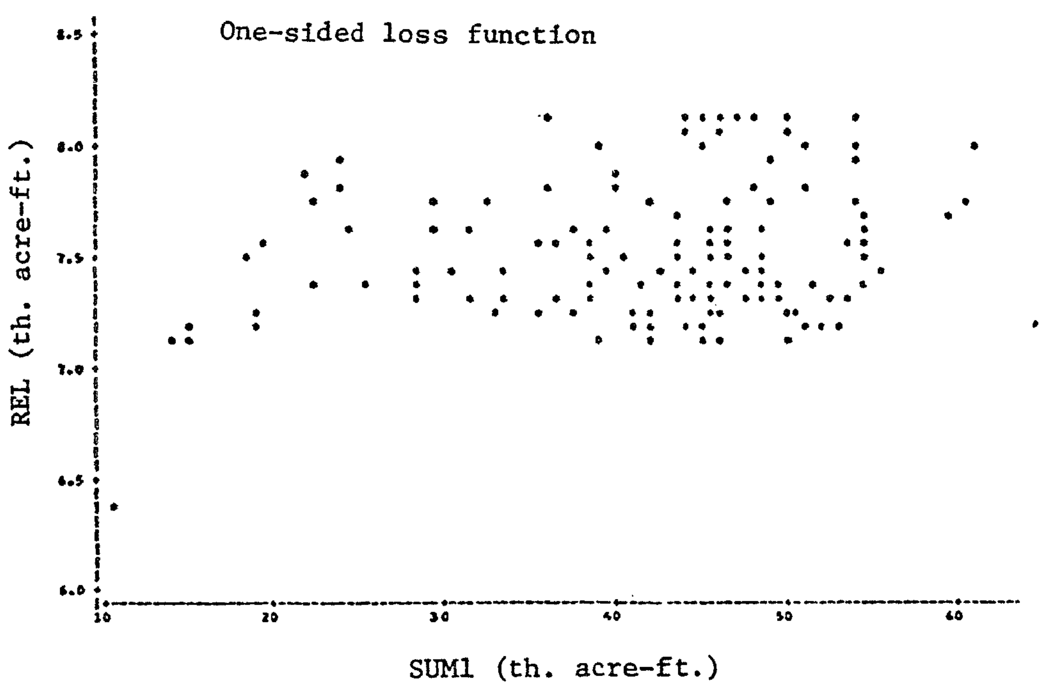
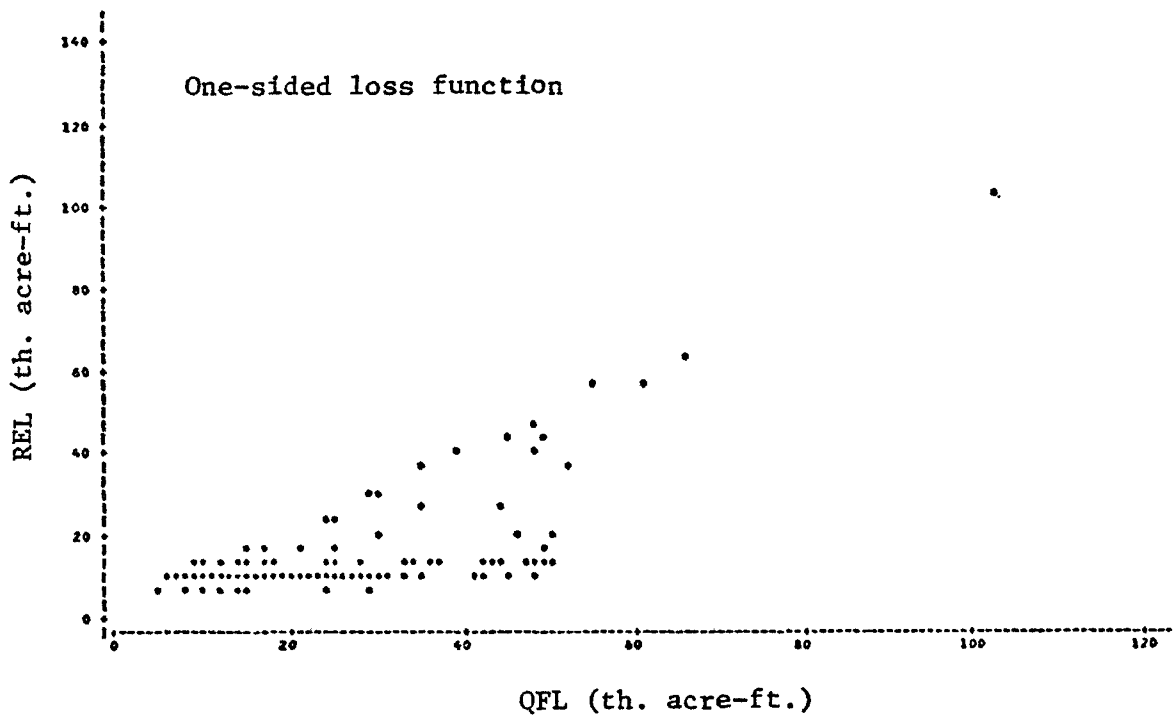


Figure G-8: Scatter Plot of Optimal Releases at Different Levels of (Inflow + Storage) in October (Target = 75 MGD)



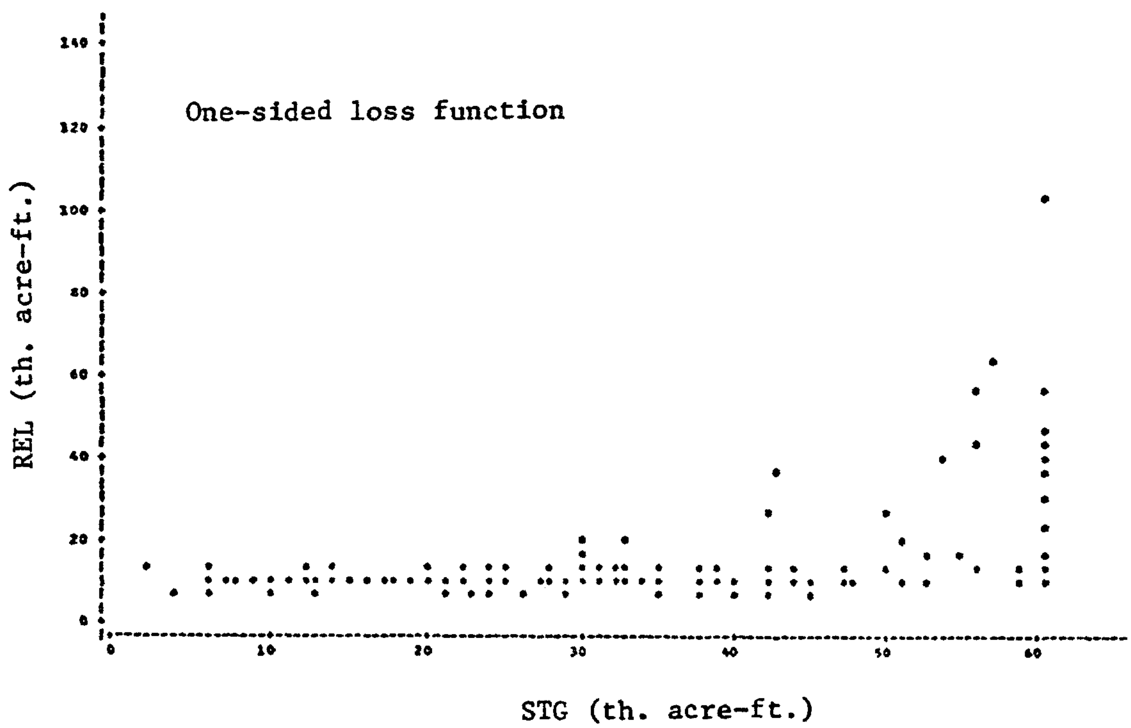


Figure G-9: Scatter Plot of Optimal Releases at Different Inflow and Storage Levels in March (Target = 120 MGD)

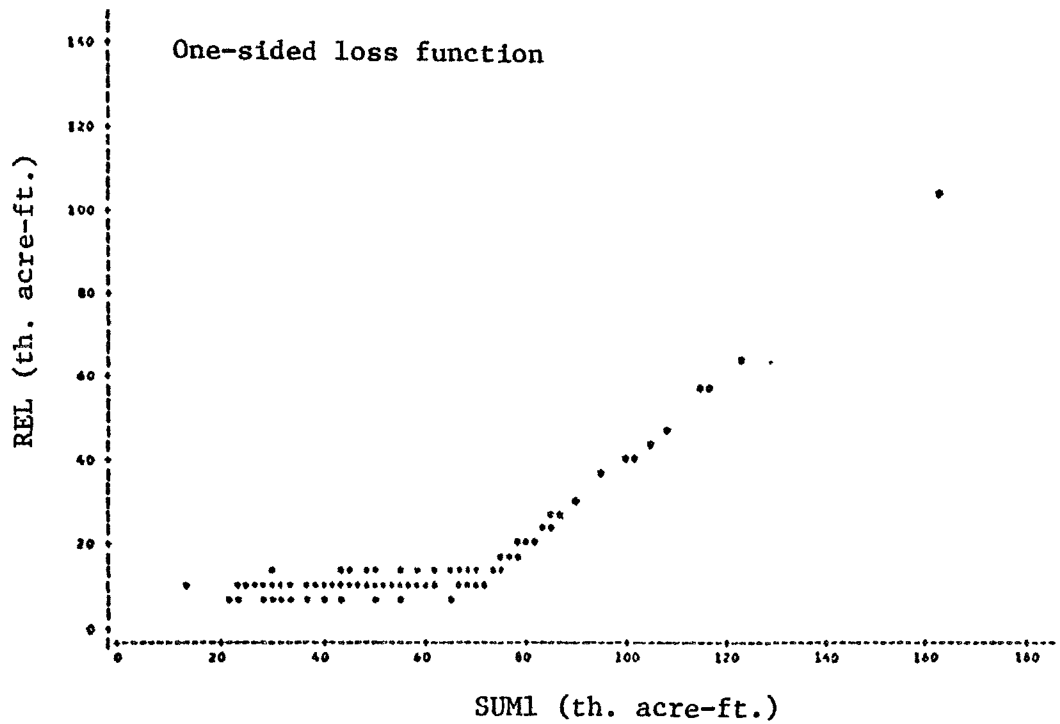
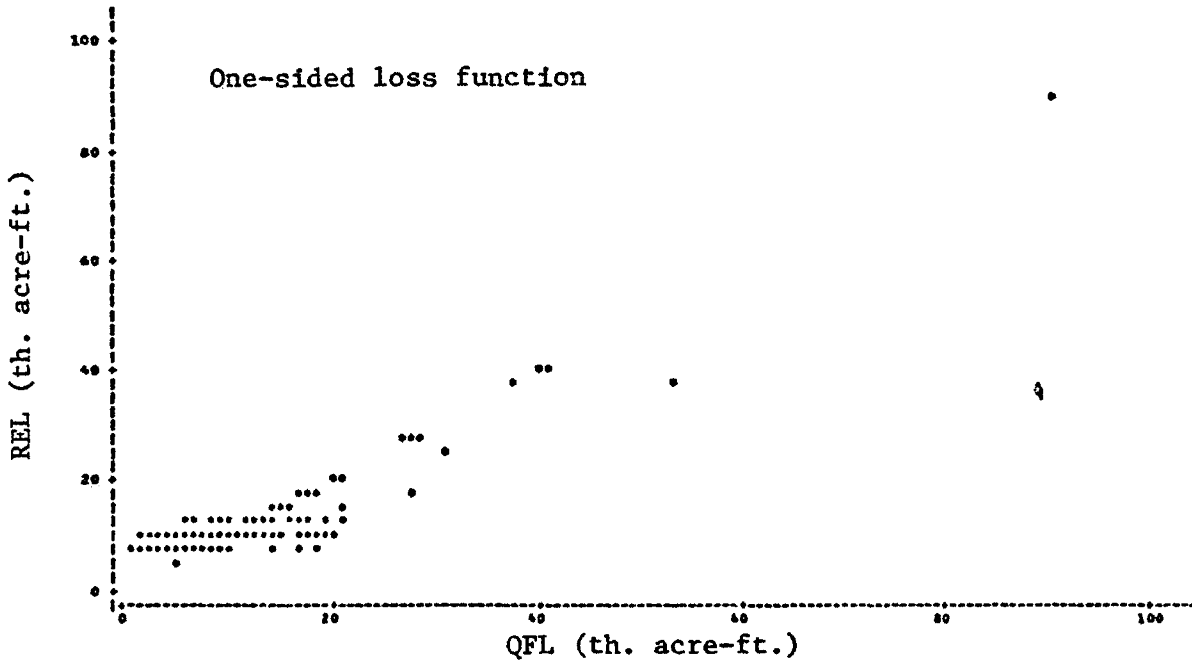


Figure G-10: Scatter Plot of Optimal Releases at Different Levels of (Inflow + Storage) in March (Target = 120 MGD)



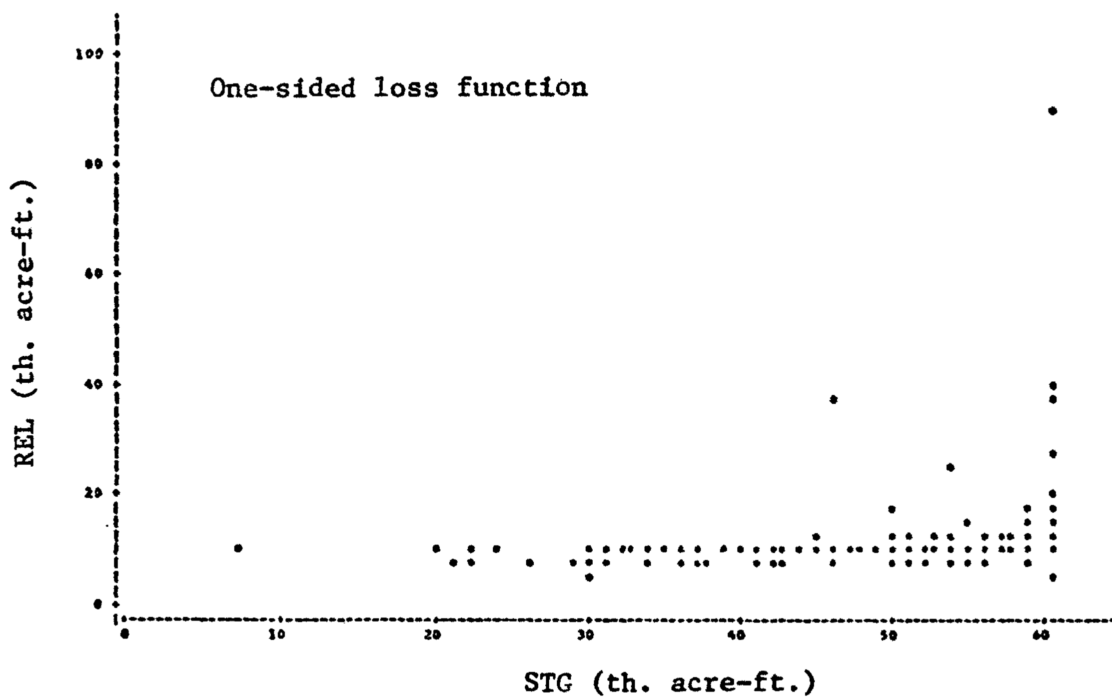


Figure G-11: Scatter Plot of Optimal Releases at Different Levels of Inflow and Storage in June (Target = 120 MGD)

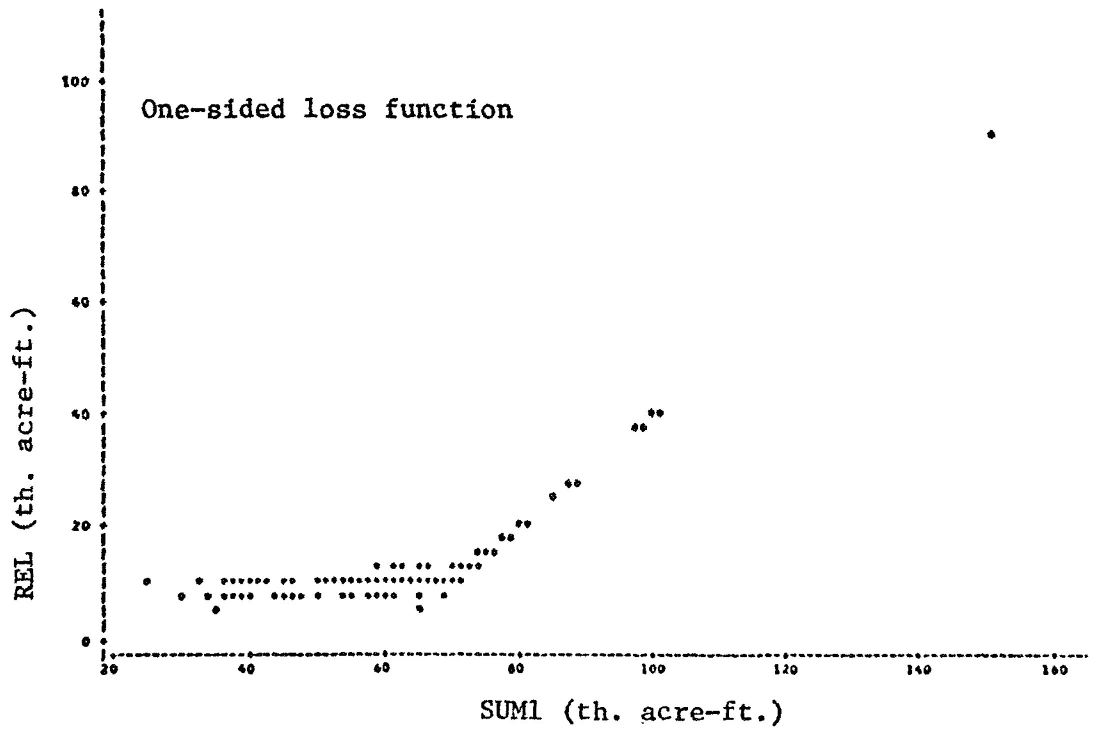
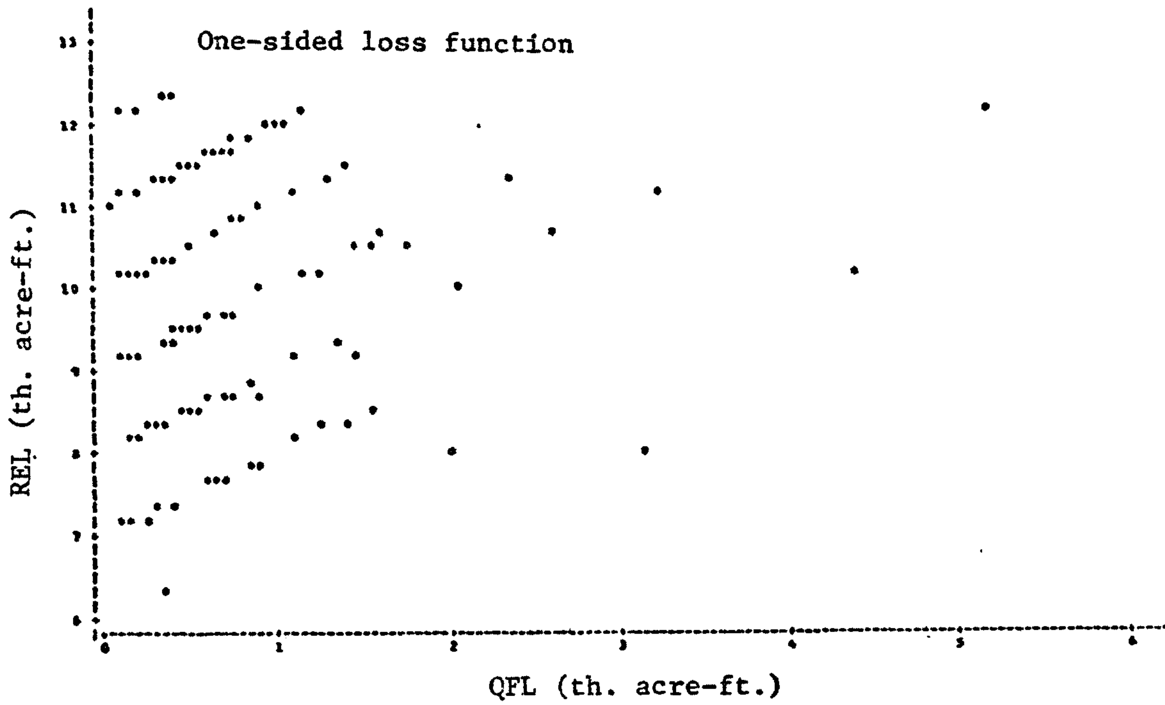


Figure G-12: Scatter Plot of Optimal Releases at Different Levels of (Inflow + Storage) in June (Target = 120 MGD)



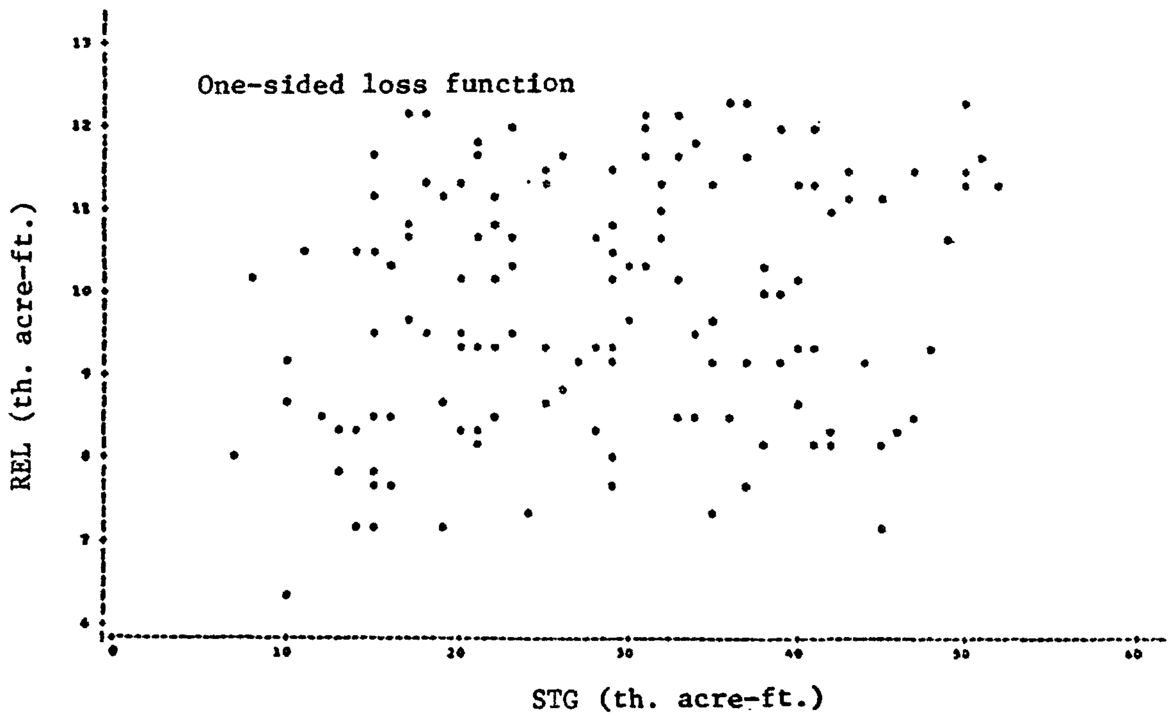


Figure G-13: Scatter Plot of Optimal Releases at Different Inflows and Storage Levels in October (Target = 120 MGD)

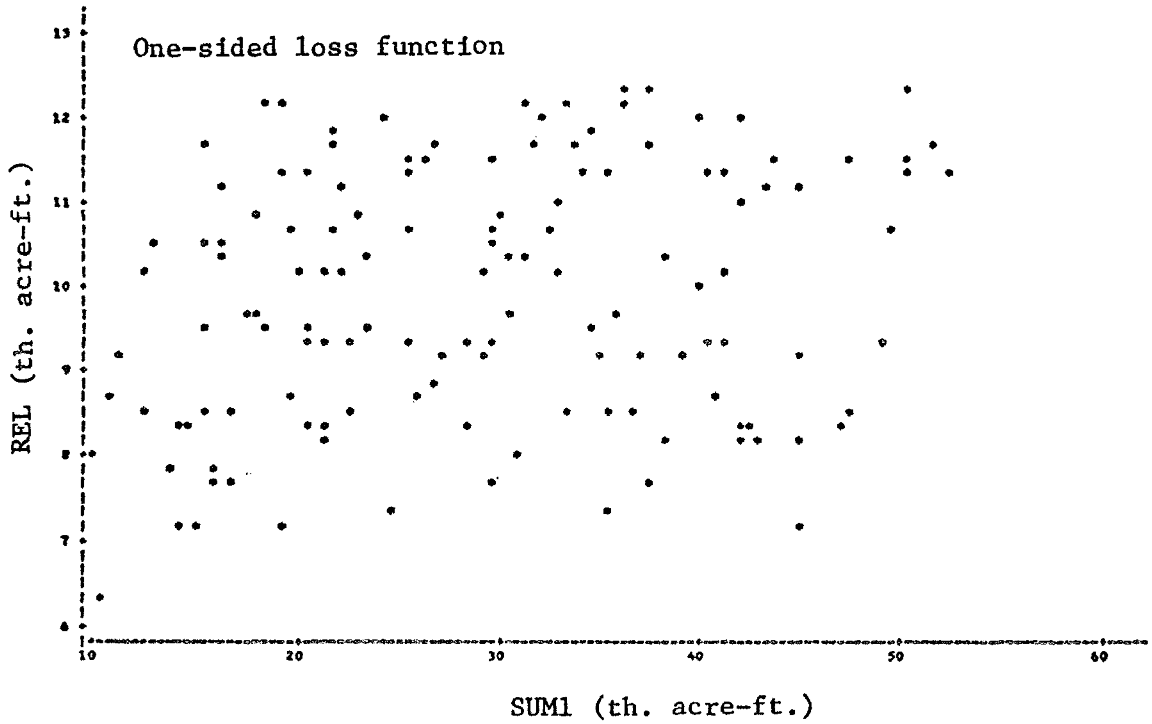


Figure G-14: Scatter Plot of Optimal Releases at Different levels of (inflow + storage) in October (Target = 120 MGD)

APPENDIX H

Optimal Monthly Release Policies Derived Using the Dynamic Programming-Regression Approach

Table H-1. Simple and complete regression policies for two-sided quadratic loss function: all target levels.

	JANUARY	R^2
Linear Model M1		
1.	REL = 2.363251 + 0.517760(QFL) (0.0235)	0.774
2.	REL = 2.363251 + 0.517760(QFL) (0.0235)	0.774
Non-Linear Model M2		
1.	REL = 8.474838 + 0.00306614(SUM2) (0.00008174)	0.908
2.	REL = 8.474838 + 0.00306614(SUM2) (0.00008174)	0.908
Non-Linear Model M3		
1.	REL = 11.019042 + 0.015900(CRP) (0.003434)	0.131

	FEBRUARY	R^2
Linear Model M1		
1.	REL = 7.513597 + 0.247908(QFL1) (0.0183)	0.565
2.	REL = 1.866474 + 0.320353(QFL) + 0.195811(QFL1) (0.0373) (0.0160)	0.715
Non-Linear Model M2		
1.	REL = 9.445002 + 0.00002913(SUM3) (0.00000203)	0.592
2.	REL = 9.445002 + 0.00002913(SUM3) (0.00000203)	0.592
Non-Linear Model M3		
1.	REL = 8.209219 + 0.011621(CRP) (0.000998)	0.489

Table H-1. (continued)

	MARCH	R^2
Linear Model M1		
1.	REL = 8.849095 + 0.208995(QFL2) (0.0171)	0.514
2.	REL = 2.631556 + 0.290600(QFL) + 0.170956(QFL2) (0.0200) (0.0111)	0.805
Non-Linear Model M2		
1.	REL = 7.960883 + 0.00002756(SUM3) (0.00000149)	0.707
2.	REL = 11.343297 - 0.00273345(SUM2) + 0.00005272(SUM3) (0.00068096) (0.00000643)	0.737
Non-Linear Model M3		
1.	REL = 6.655599 + 0.011066(CRP) (0.00092)	0.503
	APRIL	
Linear Model M1		
1.	REL = 5.364113 + 0.307617(QFL1) (0.0247)	0.521
2.	REL = 2.095789 + 0.167231(QFL) +0.195267(QFL1) + 0.110411(QFL3) (0.0238) (0.0193) (0.0094)	0.794
Non-Linear Model M2		
1.	REL = 5.894655 + 0.00002640(SUM3) (0.00000145)	0.699
2.	REL = 13.580951 - 0.00605630(SUM2) + 0.00008582(SUM3) (0.00066232) (0.00000660)	0.811
Non-Linear Model M3		
1.	REL = 4.726368 + 0.010552(CRP) (0.00069)	0.623

Table H-1. (continued)

	MAY	R^2
Linear Model M1		
1.	REL = 6.343307 + 0.463377(QFL)	0.698
	(0.0256)	
2.	REL = 4.258403 + 0.463322(QFL) + 0.085371(QFL2)	<u>0.751</u>
	(0.0233) (0.0156)	
Non-Linear Model M2		
1.	REL = 5.677940 + 0.00002619(SUM3)	0.366
	(0.00000289)	
2.	REL = 17.195966 - 0.01096047(SUM2) + 0.00014801(SUM3)	<u>0.519</u>
	(0.00163233) (0.00001832)	
Non-Linear Model M3		
1.	REL = 8.245084 + 0.007059(CRP)	0.165

	JUNE	R^2
Linear Model M1		
1.	REL = 5.930544 + 0.440285(QFL)	0.767
	(0.0204)	
2.	REL = 5.930544 + 0.440285(QFL)	0.767
	(0.0204)	
Non-Linear Model M2		
1.	REL = 5.091870 + 0.00002595(SUM3)	0.759
	(0.00000123)	
2.	REL = 11.038802 - 0.00410801(SUM2) + 0.00006402(SUM3)	<u>0.839</u>
	(0.00049228 (0.00000467)	
Non-Linear Model M3		
1.	REL = 5.595545 + 0.010550(CRP)	0.635
	(0.000671)	

Table H-1. (continued)

JULY

Linear Model M1

1. REL = 7.497293 + 0.504763(QFL) 0.621
(0.0331)

2. REL = 6.759820 + 0.321734(QFL) + 0.159839(QFL2) 0.715
(0.0394) (0.0235)

Non-Linear Model M2

1. REL = 6.792351 + 0.00002053(SUM3) 0.509
(0.00000169)

2. REL = -21.068415 + 1.833367(SUM1) - 0.03650652(SUM2)
(0.3692) (0.00628379)

+ 0.00024407(SUM3) 0.698
(0.00003404)

Non-Linear Model M3

1. REL = 7.663314 + 0.009780(CRP) 0.678
(0.000566)

AUGUST

Linear Model M1

1. REL = 8.508290 + 0.28295(QFL1) 0.410
(0.0285)

2. REL = 8.094701 + 0.180349(QFL1) + 0.089641(QFL3) 0.472
(0.0370) (0.0220)

Non-Linear Model M2

1. REL = 8.292623 + 0.00001590(SUM3) 0.347
(0.00000183)

2. REL = -15.402012 + 1.676765(SUM1) - 0.03632759(SUM2)
(0.2691) (0.00533223)

+ 0.00025926(SUM3) 0.550
(0.00003356)

Non-Linear Model M3

1. REL = 9.110315 + 0.008274(CRP) 0.487
(0.000712)

Table H-1. (continued)

SEPTEMBER

Linear Model M1

	Equation	
1.	REL = 8.912192 + 0.156941(QFL2) (0.0273)	0.189
2.	REL = 8.912192 + 0.156941(QFL2) (0.0273)	0.189

Non-Linear Model M2

	Equation	
1.	REL = 8.145286 + 0.045487(SUM1) (0.0157)	0.056
2.	REL = 8.145286 + 0.045487(SUM1) (0.0157)	0.056

Non-Linear Model M3

	Equation	
1.	REL = 9.505160 + 0.006937(CRP) (0.002405)	0.055

OCTOBER

Linear Model M1

	Equation	
1.	REL = 9.09530 + 0.163829(QFL3) (0.0270)	0.206
2.	REL = 8.654200 + 0.548497(QFL) + 0.170124(QFL3) (0.2105) (0.0266)	0.242

Non-Linear Model M2

	Equation	
1.	REL = 9.566241 + 0.017752(SUM1) (0.0168)	0.008
2.	REL = 6.814360 + 0.229103(SUM1) - 0.00352702(SUM2) (0.0984) (0.00161837)	0.040

Non-Linear Model M3

	Equation	
1.	REL = 9.677042 + 0.021587 (CRP) (0.010666)	0.028

Table H-1. (continued)

NOVEMBER

Linear Model M1

1. REL = 8.098793 + 0.481882(QFL) — 0.496
(0.0408)

2. REL = 7.355139 + 0.465331(QFL) + 0.317055(QFL4) — 0.595
(0.0368) (0.0234)

Non-Linear Model M2

1. REL = 10.247650 - 0.00000716(SUM3) — 0.003
(0.00001040)

2. REL = 5.874432 + 0.393945(SUM1) - 0.00773911(SUM2) — 0.068
(0.1235) (0.00241446)

Non-Linear Model M3

1. REL = 9.454924 + 0.011843(CRP) — 0.049
(0.004372)

DECEMBER

Linear Model M1

1. REL = 5.497170 + 0.595104(QFL) — 0.792
(0.0256)

2. REL = 5.497170 + 0.595104(QFL) — 0.792
(0.0256)

Non-Linear Model M2

1. REL = 7.979547 + 0.00581901(SUM2) — 0.636
(0.00036921)

2. REL = 7.979547 + 0.00581901(SUM2) — 0.636
(0.00036921)

Non-Linear Model M3

1. REL = 11.539973 + 0.009897(CRP) — 0.011
(0.007740)

*Numbers in parentheses represent the standard error associated with the regression coefficients.

Table H-2. Simple and Complete Regression Policies for One-Sided Quadratic Loss Function: 75 MGD. Target

JANUARY

Linear Model M1	R^2
1. REL = -3.422078 + 0.805268(QFL) (0.0296)	0.839
2. REL = -3.422078 + 0.805268(QFL) (0.0296)	<u>0.839</u>
Non-Linear Model M2	
1. REL = 2.714038 + 0.00293737(SUM2) (0.0000593)	0.945
2. REL = 2.714038 + 0.00293737(SUM2) (0.0000593)	<u>0.945</u>
Non-Linear Model M3	
1. REL = 1.350107 + 0.015894(CRP) (0.000317)	<u>0.947</u>

FEBRUARY

Linear Model M1	
1. REL = -2.253630 + 0.751493(QFL) (0.0543)	0.574
2. REL = -12.184527 + 0.642622(QFL) + 0.301707(STG) (0.0427) (0.0296)	<u>0.755</u>
Non-Linear Model M2	
1. REL = 2.434284 + 0.00003395(SUM3) (0.00000073)	0.939
2. REL = 2.434284 + 0.00003395(SUM3) (0.00000073)	<u>0.939</u>
Non-Linear Model M3	
1. REL =-0.319151 + 0.015409(CRP) 0.000437)	<u>0.898</u>

Table H-2. (continued)

MARCH

Linear Model M1

1. REL = $-2.588792 + 0.892509(QFL)$ — 0.772
   (0.0407)

2. REL = $-17.868505 + 0.820891(QFL) + 0.353249(STG)$ — 0.880
   (0.0303) (0.0314)

Non-Linear Model M2

1. REL = $-4.447540 + 0.00407779(SUM2)$ — 0.926
   (0.0000970)

2. REL = $-4.447540 + 0.00407779(SUM2)$ — 0.926
   (0.0000970)

Non-Linear Model M3

1. REL = $-0.460003 + 0.016083(CRP)$ — 0.957
   (0.000288)

APRIL

Linear Model M1

1. REL = $-2.170014 + 0.979618(QFL)$ — 0.844
   (0.0354)

2. REL = $-18.232745 + 0.915497(QFL) + 0.325466(STG)$ — 0.928
   (0.0246) (0.0253)

Non-Linear Model M2

1. REL = $-0.997323 + 0.00003552(SUM3)$ — 0.954
   (0.00000066)

2. REL = $-0.997323 + 0.00003552(SUM3)$ — 0.954
   (0.00000066)

Non-Linear Model M3

1. REL = $-0.619128 + 0.016515(CRP)$ — 0.967
   (0.000257)

Table H-2. (continued)

MAY

Linear Model M1

1.	REL = 3.226631 + 0.762783(QFL) (0.0195)	0.915
2.	REL = 3.226631 + 0.762783(QFL) (0.0195)	0.915

Non-Linear Model M2

1.	REL = 0.696137 + 0.00003275(SUM3) (0.000001)	0.883
2.	REL = 0.696137 + 0.00003275(SUM3) (0.000001)	0.883

Non-Linear Model M3

1.	REL = 2.516191 + 0.014722(CRP) (0.000304)	0.943

JUNE

Linear Model M1

1.	REL = 1.416870 + 0.906205(QFL) (0.0220)	0.923
2.	REL = 1.416870 + 0.906205(QFL) (0.0220)	0.923

Non-Linear Model M2

1.	REL = 1.914207 + 0.00002927(SUM3) (0.00000078)	0.908
2.	REL = 1.914207 + 0.00002927(SUM3) (0.00000078)	0.908

Non-Linear Model M3

1.	REL = 2.017478 + 0.015042(CRP) (0.000276)	0.954

Table H-2. (continued)

JULY

Linear Model M1

1. REL = 4.591349 + 0.710013(QFL) — 0.825
   (0.0274)

2. REL = 4.591349 + 0.710013(QFL) — <u>0.825</u>
   (0.0274)

Non-Linear Model M2

1. REL = 1.655222 + 0.00002848(SUM3) — 0.696
   (0.00000158)

2. REL = 15.520785 - 0.00998620(SUM2) + 0.00012686(SUM3) — <u>0.945</u>
   (0.00039619) (0.00000396)

Non-Linear Model M3

1. REL = 4.764758 + 0.011954(CRP) — <u>0.853</u>
   (0.000416)

AUGUST

Linear Model M1

1. REL = 6.276874 + 0.624624(QFL) — 0.790
   (0.0270)

2. REL = 6.276874 + 0.624624(QFL) — <u>0.790</u>
   (0.0270)

Non-Linear Model M2

1. REL = 3.544357 + 0.00002430(SUM3) — 0.636
   (0.00000154)

2. REL = 12.765413 - 0.00745992(SUM2) + 0.0001009(SUM3) — <u>0.927</u>
   (0.0003135) (0.00000329)

Non-Linear Model M3

1. REL = 6.326731 + 0.011060(CRP) — <u>0.835</u>
   (0.000412)

Table H-2. (continued)

SEPTEMBER

Linear Model M1

1. REL = 7.288852 + 0.014318(QFL1) — 0.061
   (0.0047)

2. REL = 7.288852 + 0.014318(QFL1) — 0.061
   (0.0047)

Non-Linear Model M2

1. REL = 7.154631 + 0.00000137(SUM3) — 0.083
   (0.00000038)

2. REL = 7.154631 + 0.00000137(SUM3) — 0.083
   (0.00000038)

Non-Linear Model M3

1. REL = 7.299834 + 0.000460(CRP) — 0.028
   (0.000226)

OCTOBER

Linear Model M1

1. REL = 7.241665 + 0.006643(STG) — 0.050
   (0.0024)

2. REL = 7.241665 + 0.006643(STG) — 0.050
   (0.0024)

Non-Linear Model M2

1. REL = 7.233419 + 0.006723(SUM1) — 0.052
   (0.002410)

2. REL = 7.233419 + 0.006723(SUM1) — 0.052
   (0.002410)

Non-Linear Model M3

1. REL = 7.485056 + 0.001011(CRP) — 0.013
   (0.000737)

Table H-2. (continued)

NOVEMBER

Linear Model M1

1. REL = 7.187989 + 0.053847(QFL) 0.177
(0.0098)

2. REL = 6.887654 + 0.050836(QFL) + 0.009057(STG) 0.211
(0.0097) (0.0036)

Non-Linear Model M2

1. REL = 7.117811 + 0.00000411(SUM3) 0.207
(0.00000067)

2. REL = 7.117811 + 0.00000411(SUM3) 0.207
(0.00000067)

Non-Linear Model M3

1. REL = 7.200533 + 0.001425(CRP) 0.226
(0.000221)

DECEMBER

Linear Model M1

1. REL = 1.744254 + 0.673905(QFL) 0.743
(0.0333)

2. REL = 1.744254 + 0.673905(QFL) 0.743
(0.0333)

Non-Linear Model M2

1. REL = 5.981962 + 0.00002309(SUM3) 0.951
(0.00000044)

2. REL = 5.981962 + 0.00002309(SUM3) 0.951
(0.00000044)

Non-Linear Model M3

1. REL = 3.694162 + 0.014104(CRP) 0.890
(0.000415)

Table H-3. Simple and Complete Regression Policies for One-Sided Quadratic Loss Function: 100 MGD Target

JANUARY

$\underline{R^2}$

Linear Model M1

1. $REL = -1.932056 + 0.681189(QFL)$ — 0.721
   (0.0356)

2. $REL = -3.741529 + 0.511924(QFL) + 0.522398(QFL1)$ — 0.783
   (0.0413) (0.0823)

Non-Linear Model M2

1. $REL = 5.871937 + 0.00276234(SUM2)$ — 0.955
   (0.00005)

2. $REL = 5.871937 + 0.00276234(SUM2)$ — 0.955
   (0.00005)

Non-Linear Model M3

1. $REL = 5.419197 + 0.015391(CRP)$ — 0.927
   (0.000363)

FEBRUARY

Linear Model M1

1. $REL = 6.338181 + 0.248235(QFL1)$ — 0.527
   (0.0198)

2. $REL = 0.814748 + 0.267061(QFL) + 0.128497(QFL1) + 0.235504(QFL2)$ — 0.722
   (0.0389) (0.0204) (0.0405)

Non-Linear Model M2

1. $REL = 5.758434 + 0.00002947(SUM3)$ — 0.913
   (0.00000076)

2. $REL = 5.758434 + 0.00002947(SUM3)$ — 0.913
   (0.00000076)

Non-Linear Model M3

1. $REL = 4.016018 + 0.012704(CRP)$ — 0.849
   (0.00045)

Table H-3. (continued)

MARCH

Linear Model M1

1. REL = -1.008429 + 0.686371(QFL) 0.579
(0.0492)

2. REL = -12.162808 + 0.640091(QFL) + 0.328149(STG) 0.761
(0.0375) (0.0317)

Non-Linear Model M2

1. REL = 6.123628 + 0.00002681(SUM3) 0.927
(0.00000063)

2. REL = 6.123628 + 0.00002681(SUM3) 0.927
(0.00000063)

Non-Linear Model M3

1. REL = 1.827556 + 0.014722(CRP) 0.924
(0.00035)

APRIL

Linear Model M1

1. REL = -1.408316 + 0.833798(QFL) 0.701
(0.0458)

2. REL = -13.718943 + 0.775105(QFL) + 0.293293(STG) 0.838
(0.0342) (0.0268)

Non-Linear Model M2

1. REL = 3.286003 + 0.00003248(SUM3) 0.941
(0.00000068)

2. REL = 3.286003 + 0.00003248(SUM3) 0.941
(0.00000068)

Non-Linear Model M3

1. REL = 0.693003 + 0.015419(CRP) 0.936
(0.00034)

Table H-3. (continued)

MAY

Linear Model M1

1. $REL = 6.445504 + \underset{(0.0281)}{0.465933}(QFL)$ — 0.659

2. $REL = -1.211953 + \underset{(0.0241)}{0.489652}(QFL) + \underset{(0.0195)}{0.144988}(STG)$ — 0.755

Non-Linear Model M2

1. $REL = 3.849741 + \underset{(0.00000118)}{0.00002797}(SUM3)$ — 0.797

2. $REL = 12.707738 - \underset{(0.00041)}{0.00658352}(SUM2) + \underset{(0.000004)}{0.0000923}(SUM3)$ — 0.929

Non-Linear Model M3

1. $REL = 5.087601 + \underset{(0.00035)}{0.011814}(CRP)$ — 0.789

JUNE

Linear Model M1

1. $REL = 2.564842 + \underset{(0.0267)}{0.834347}(QFL)$ — 0.873

2. $REL = 2.564842 + \underset{(0.0267)}{0.834347}(QFL)$ — 0.873

Non-Linear Model M2

1. $REL = 4.335639 + \underset{(0.00000068)}{0.00002716}(SUM3)$ — 0.919

2. $REL = 4.335639 + \underset{(0.00000068)}{0.00002716}(SUM3)$ — 0.919

Non-Linear Model M3

1. $REL = 3.663343 + \underset{(0.000308)}{0.014171}(CRP)$ — 0.937

Table H-3. (continued)

JULY

Linear Model M1

1.	REL = 6.375888 + 0.634177(QFL) (0.0292)	0.768
2.	REL = 6.375888 + 0.634177(QFL) (0.0292)	0.768

Non-Linear Model M2

1.	REL = 5.328408 + 0.00002328(SUM3) (0.00000141)	0.656
2.	REL = 14.227359 - 0.007527(SUM2) + 0.00010101(SUM3) (0.0004275) (0.00000449)	0.893

Non-Linear Model M3

1.	REL = 6.745824 + 0.010871(CRP) (0.00043)	0.820

AUGUST

Linear Model M1

1.	REL = 8.211463 + 0.481441(QFL) (0.0348)	0.574
2.	REL = 8.211463 + 0.481441(QFL) (0.0348)	0.574

Non-Linear Model M2

1.	REL = 6.668898 + 0.00002106(SUM3) (0.0000014)	0.614
2.	REL = -9.234424 + 1.247091(SUM1) - 0.027521(SUM2) + 0.00019926(SUM3) (0.1217) (0.00221) (0.00001241)	0.908

Non-Linear Model M3

1.	REL = 8.131475 + 0.010635(CRP) (0.000474)	0.780

Table H-3. (continued)

SEPTEMBER

Linear Model M1

1. REL = 7.950423 + 0.025391(STG) 0.100
(0.0064)

2. REL = 7.950423 + 0.025391(STG) 0.100
(0.0064)

Non-Linear Model M2

1. REL = 7.807141 + 0.027995(SUM1) 0.121
(0.0063)

2. REL = 7.807141 + 0.027995(SUM1) 0.121
(0.0063)

Non-Linear Model M3

1. REL = 8.815095 + 0.002384(CRP) 0.043
(0.000941)

OCTOBER

Linear Model M1

1. REL = 8.393049 + 0.028395(STG) 0.108
(0.0068)

2. REL = 8.393049 + 0.028395(STG) 0.108
(0.0068)

Non-Linear Model M2

1. REL = 8.327370 + 0.029775(SUM1) 0.117
(0.0069)

2. REL = 8.327370 + 0.029775(SUM1) 0.117
(0.0069)

Non-Linear Model M3

1. REL = 9.065944 + 0.010367(CRP) 0.057

Table H-3. (continued)

NOVEMBER

Linear Model M1

1. REL = 8.640208 + 0.095683(QFL) 0.145
(0.0195)

2. REL = 8.039696 + 0.105909(QFL) + 0.024085(STG) 0.220
(0.0189) (0.0065)

Non-Linear Model M2

1. REL = 8.233236 + 0.029565(SUM1) 0.119
(0.0068)

2. REL = 8.233236 + 0.029565(SUM1) 0.119
(0.0068)

Non-Linear Model M3

1. REL = 8.738685 + 0.003328(CRP) 0.142

DECEMBER

Linear Model M1

1. REL = 5.408182 + 0.423291(QFL) 0.553
(0.0319)

2. REL = 5.408182 + 0.423291(QFL) 0.553
(0.0319)

Non-Linear Model M2

1. REL = 8.611039 + 0.00002216(SUM3) 0.967
(0.00000035)

2. REL = 8.611039 + 0.00002216(SUM3) 0.967
(0.00000035)

Non-Linear Model M3

1. REL = 6.992027 + 0.014529(CRP) 0.872
(0.00047)

Table H-4. Simple and Complete Regression Policies for One-Sided Quadratic Loss-Function: 120-MGD Target

JANUARY

$\underline{R^2}$

Linear Model M1

1. $REL = -1.088610 + \underset{(0.0367)}{0.650983}(QFL)$ — 0.689

2. $REL = -2.899127 + \underset{(0.0429)}{0.481621}(QFL) + \underset{(0.0856)}{0.522700}(QFL1)$ — $\underline{0.754}$

Non-Linear Model M2

1. $REL = 7.318042 + \underset{(0.000043)}{0.002740}(SUM2)$ — 0.966

2. $REL = 7.318042 + \underset{(0.000043)}{0.002740}(SUM2)$ — $\underline{0.966}$

Non-Linear Model M3

1. $REL = 7.567831 + \underset{(0.000310)}{0.015557}(CRP)$ — 0.946

FEBRUARY

Linear Model M1

1. $REL = 6.188818 + \underset{(0.0158)}{0.228083}(QFL1)$ — 0.595

2. $REL = 1.946024 + \underset{(0.0297)}{0.194856}(QFL) + \underset{(0.0156)}{0.120820}(QFL1) + \underset{(0.0309)}{0.233244}(QFL2)$ — $\underline{0.783}$

Non-Linear Model M2

1. $REL = 6.857322 + \underset{(0.00000082)}{0.00002789}(SUM3)$ — 0.890

2. $REL = 10.064991 - \underset{(0.000267)}{0.003318}(SUM2) + \underset{(0.00000252)}{0.00005852}(SUM3)$ — $\underline{0.948}$

Non-Linear Model M3

1. $REL = 5.526906 + \underset{(0.000515)}{0.011423}(CRP)$ — 0.776

Table H-4. (continued)

MARCH

Linear Model M1

1.	REL = -0.111666 + 0.611772(QFL) (0.0432)	0.531
2.	REL = -9.926625 + 0.580123(QFL) + 0.318793(STG) (0.0374) (0.0325)	0.722

Non-Linear Model M2

1.	REL = 7.002913 + 0.00002578(SUM3) (0.00000061)	0.926
2.	REL = 7.002913 + 0.00002578(SUM3) (0.00000061)	0.926

Non-Linear Model M3

1.	REL = 3.040460 + 0.014145(CRP) (0.000386)	0.904

APRIL

Linear Model M1

1.	REL = 0.204264 + 0.720240(QFL) (0.0493)	0.600
2.	REL = -12.276322 + 0.681662(QFL) + 0.300613(STG) (0.0375) (0.0291)	0.773

Non-Linear Model M2

1.	REL = 4.112362 + 0.00003122(SUM3) (0.00000072)	0.930
2.	REL = 4.112362 + 0.00003122(SUM3) (0.00000072)	0.930

Non-Linear Model M3

1.	REL = 1.316309 + 0.014801(CRP) (0.000403)	0.905

Table H-4. (continued)

MAY

Linear Model M1

1. REL = 7.273912 + 0.383093(QFL) 0.566
   (0.0281)

2. REL = -1.046410 + 0.430359(QFL) + 0.159478(STG) <u>0.720</u>
   (0.0233) (0.0181)

Non-Linear Model M2

1. REL = 4.860521 + 0.00002662(SUM3) 0.780
   (0.00000119)

2. REL = 13.002950 -0.006385(SUM2) + 0.00009064(SUM3) <u>0.911</u>
   (0.000442) (0.0000045)

Non-Linear Model M3

1. REL = 5.888491 + 0.010923(CRP) 0.707
   (0.000590)

JUNE

Linear Model M1

1. REL = 2.958939 + 0.771181(QFL) 0.824
   (0.0299)

2. REL = 2.958939 + 0.771181(QFL) <u>0.824</u>
   (0.0299)

Non-Linear Model M2

1. REL = 4.929130 + 0.00002632(SUM3) 0.916
   (0.00000067)

2. REL = 4.929130 + 0.00002632(SUM3) <u>0.916</u>
   (0.00000067)

Non-Linear Model M3

1. REL = 4.031915 + 0.013642(CRP) 0.907
   (0.000366)

Table H-4. (continued)

JULY

Linear Model M1

1. REL = 6.965086 + 0.610118(QFL) 0.709
   (0.0328)

2. REL = 6.413782 + 0.473293(QFL) + 0.119489(QFL2) 0.750
   (0.0417) (0.0248)

Non-Linear Model M2

1. REL = 6.167874 + 0.00002314(SUM3) 0.644
   (0.00000144)

2. REL = -23.748743 + 2.028723(SUM1) - 0.040804(SUM2) + 0.00027208(SUM3) 0.879
   (0.2665) (0.004502) (0.00002409)

Non-Linear Model M3

1. REL = 8.707438 + 0.0101844(CRP) 0.737
   (0.000544)

AUGUST

Linear Model M1

1. REL = 8.752117 + 0.481717(QFL) 0.528
   (0.0382)

2. REL = 5.982533 + 0.466619(QFL) + 0.064412(STG) 0.574

Non-Linear Model M2

1. REL = 7.423871 + 0.0000219(SUM3) 0.603
   (0.00000149)

2. REL = -8.360040 + 1.198246(SUM1) - 0.026154(SUM2) + 0.00018967(SUM3) 0.822
   (0.1729) (0.003159) (0.000018)

Non-Linear Model M3

1. REL = 8.707438 + 0.010844(CRP) 0.737
   (0.000544)

Table H-4. (continued)

SEPTEMBER

Linear Model M1

1.	REL = 8.934620 + 0.091923(QFL2) (0.0195)	0.135
2.	REL = 8.934620 + 0.091923(QFL2) (0.0195)	0.135

Non-Linear Model M2

1.	REL = 8.144735 + 0.035144(SUM1) (0.0105)	0.073
2.	REL = 8.144735 + 0.035144(SUM1) (0.0105)	0.073

Non-Linear Model M3

1.	REL = 9.223543 + 0.005112(CRP) (0.001602)	0.067

OCTOBER

Linear Model M1

1.	REL = 9.282482 + 0.095262(QFL3)	0.141
2.	REL = 9.282482 + 0.095262(QFL3) (0.0197)	0.141

Non-Linear Model M2

1.	REL = 9.091869 + 0.025838(SUM1) (0.011346)	0.035
2.	REL = 9.091869 + 0.025838(SUM1) (0.011346)	0.035

Non-Linear Model M3

1.	REL = 9.519953 + 0.017140(CRP) (0.006408)	0.048

Table H-4. (continued)

NOVEMBER

Linear Model M1

1. REL = 8.785439 + 0.199551(QFL) 0.244
(0.0295)

2. REL = 8.321816 + 0.189233(QFL) + 0.085446(QFL4) 0.355
(0.0274) (0.0174)

Non-Linear Model M2

1. REL = 9.041920 + 0.024619(SUM1) 0.027
(0.012300)

2. REL = 9.041920 + 0.024619(SUM1) 0.027
(0.012300)

Non-Linear Model M3

1. REL = 9.138546 + 0.007047(CRP) 0.143
(0.001448)

DECEMBER

Linear Model M1

1. REL = 6.730518 + 0.346617(QFL) 0.496
(0.0293)

2. REL = 6.730518 + 0.346617(QFL) 0.496
(0.0293)

Non-Linear Model M2

1. REL = 9.483786 + 0.00002305(SUM3) 0.932
(0.00000052)

2. REL = 9.483786 + 0.00002305(SUM3) 0.932
(0.00000052)

Non-Linear Model M3

1. REL = 8.317418 + 0.015899(CRP) 0.834
(0.000595)

APPENDIX I

Computer Programs

Description of Computer Programs: The computer programs listed in this Appendix are written in FORTRAN-IV Language and require a FORTRAN-IV compiler. A source deck may be obtained from the Department of Civil Engineering, The Ohio State University. The following table summarizes relevant information, when the programs are compiled and executed on an IBM 370/168 computer series.

Table I-1: Computer Requirements

Computer Program Description	Storage* Required (Bytes)	CPU Time (seconds)	Total Cost per Run (dollars)
1. Dynamic Program	650 K	35	10.00
2. Simulation of Reservoir Operation	250 K	15	5.00

*K represents 1000 bytes.

Solution of Dynamic Programming Algorithm

```
// TIME=2,REGION=900K
/*JOBPARM LINES=3000,DISKIO=5000
// EXEC FORTGCG,REGION=900K,TIME=2,
// PARM.GO='LET,MAP,NOPLS,PRINT,SIZE=500K'
//CMP.SYSIN DD *
      COMMON R(1000),NS
      DIMENSION QK(1000)
      DEFINE FILE 9(600,600,U,IFL)
      IFL=16
C  INPUT THE MONTHLY INFLOWS EXPRESSED IN TH. AC.FT. PER MONTH.
C  THE INFLOWS ARE RE-ARRANGED FOR SOLVING A BACKWARD -RECURSIVE
C  DYNAMIC PROGRAM,WHITCH CONSISTS OF 600 STAGES OR 50 YEARS
C  GENERATED INFLOWS.
      WRITE(6,701)
  701 FORMAT(///30X,'GENERATED FLOWS IN TH.AC.FT./MT.',5X///)
      NS=600
      READ(9'IFL) (QK(NT),NT=1,NS)
      K=NS+1
      DO 40 J=1,NS
      K=K-1
      R(K)=QK(J)
   40 CONTINUE
      WRITE(6,75) (R(K),K=1,NS)
   75 FORMAT(10X,6F15.4/10X,6F15.4)
      CALL DYPGM
      STOP
      END
      SUBROUTINE DYPGM
      COMMON R(1000),NS
      DIMENSION S(650,65),X(65),XOPT(650,65),STOG(650),RET(65),OBJ(65),
     1OBJ1(65),XT(650),W(55,12),Z1(55,12),Y1(55,12),NSHT(55),NSTA(650),
     2VAR(75,25),ND(12),XT1(12),XTAR(650)
C READ INPUT PARAMETERS.
      READ(5,1)CAP,SFS,SMIN,XPT,GROS,KS
    1 FORMAT(5F10.0,I5)
      READ(5,906) (ND(J),J=1,12)
  906 FORMAT(12I5)
      DO 31 J=1,12
      TT=XPT/0.326
      TT1=(TT*ND(J))/1000.0
      XT1(J)=TT1
   31 CONTINUE
      J=13
      DO 41 I=1,NS
      J=J-1
      XTAR(I)=XT1(J)
      IF(J.EQ.1) J=13
   41 CONTINUE
      WRITE(6,42) (XTAR(I),I=1,NS)
   42 FORMAT(//'MONTHLY TARGET REL. IN TH.AC.FT.'/(12F8.4))
C SOLVE D.P. USING BACKWARD RECURSSION STARTING WITH THE LAST STAGE
C AS STAGE 1.
C SOLUTION AT STAGE 1.
      KC=KS
C SET THE ASSIGNED GRID POINT STORAGE WHITCH ARE ASSUMED TO BE THE
C SAME FOR ALL STAGES.
C SET THE INITIAL STORGE LEVELS WHERE APPROPRIATE.
      SMAX=CAP-SFS
      DO 11 N=1,NS
      S(N,1)=SMIN
      S(N,KS)=SMAX
      S(N,2)=2.5
```

```
      S(N,KS-1)=27.0
   11 CONTINUE
      DO 2 N=1,NS
      K1=KS-1
      DO 2 K=3,K1
      S(N,K)=S(N,K-1) + GROS
    2 CONTINUE
C SET THE END STORAGE LEVEL.
      SEND=SMIN
      N=1
      K=1
  200 X(K)= S(N,K) + R(N) -SEND
      IF(X(K).GE.0.0) GO TO 3
      K=K+1
      GO TO 200
    3 IF(K.LT.KC)KC=K
      RET(K)= (X(K)-XTAR(N))**2
C COMPUTE THE OPTIMAL RELEASES AND RETURNS AT STAGE 1 FOR
C  STORAGE INPUT S(N,K).
      OBJ(K)=RET(K)
      XOPT(N,K)=X(K)
      IF(K.EQ.KS) GO TO 4
      K=K+1
      GO TO 200
    4 DO 5 I=KC,KS
      OBJ1(I)=OBJ(I)
    5 CONTINUE
C SOLVE FOR THE REMAINING STAGES STARTING WITH STAGE 1 .
      N=2
  300 K=1
C SET THE INITIAL VALUE OPTIMAL RETURNS OBJ(K) TO A HIGH VALUE.
      DO 6 I=1,KS
      OBJ(I)=10.0**8
    6 CONTINUE
  400 K1=1
      IF(N.EQ.2) K1=KC
      SUM= S(N,K) + R(N)
      IF(SUM.GT.SMAX) GO TO 500
  450 X(K) = S(N,K) + R(N)-S(N-1,K1)
      IF(X(K).GE.0.0) GO TO 7
      K=K+1
      GO TO 400
C  COMPUTE THE RETURNS OF THE FEASIBLE RELEASE
    7 SUM4=(X(K)-XTAR(N))**2
      RET(K)= SUM4 + OBJ1(K1)
      IF(RET(K).LE.OBJ(K)) XOPT(N,K)= X(K)
      IF(RET(K).LE.OBJ(K)) OBJ(K)=RET(K)
      IF(K1.EQ.KS) GO TO 8
      K1=K1 +1
      GO TO 450
    8 IF(K.EQ.KS) GO TO 9
      K=K+1
      GO TO 400
    9 IF(N.GT.2) KC=1
      DO 10 I=KC,KS
      OBJ1(I)= OBJ(I)
   10 CONTINUE
      N=N+1
      IF(N.EQ.NS) GO TO 800
      GO TO 300
  500 K1=KS
  525 X(K)= S(N,K) + R(N) -S(N-1,K1)
```

```
      IF(X(K).GE.0.0) GO TO 550
      K=K+1
      GO TO 400
C COMPUTE THE RETURNS OF THE FEASIBLE RELEASES.
  550 SUM5=(X(K)-XTAR(N))**2
      RET(K)=SUM5 + OBJ1(K1)
      IF(RET(K).LE.OBJ(K)) XOPT(N,K)= X(K)
      IF(RET(K).LE.OBJ(K)) OBJ(K)=RET(K)
      IF(N.EQ.2.AND.K1.EQ.KC) GO TO 575
      IF(K1.EQ.1) GO TO 575
      K1=K1-1
      GO TO 525
  575 IF(K.EQ.KS) GO TO 576
      K=K+1
      GO TO 500
  576 IF(N.EQ.NS) GO TO 800
      DO 69 K=1,KS
      OBJ1(K)=OBJ(K)
   69 CONTINUE
      N=N+1
      GO TO 300
C TRACE THE OPTIMAL PATH FOR A GIVEN INPUT.
  800 WRITE(6,55)
   55 FORMAT(///1X,'STAGE',1X,'STATE',20X,'OPT.REL.-STG.N',5X,
     1'OPT.RET.-STG.N   ',10X,'OPT.RET.-STG.N-1'//)
      DO 13 K=1,KS
      T5=XOPT(N,K)
      T6=OBJ(K)
      T7=OBJ1(K)
      WRITE(6,51)N,K,T5,T6,T7
   51 FORMAT(/2I5,20X,2F15.4,20X,F15.4)
   13 CONTINUE
      K=KS
      NSTA(600)=K
      STOG(NS)=S(NS,K)
      N=NS
C DETERMINE THE END OPTIMAL STOARGE AT PERIOD N.
  850 ST=S(N,K) + R(N)-XOPT(N,K)
      XT(N)=XOPT(N,K)
      K2=1
  860 DIFF=(ST-S(N-1,K2))
      IF(DIFF.LT.0.00001) GO TO 870
      SP=S(N-1,K2)
      K2=K2 +1
      GO TO 860
  870 ST=S(N-1,K2)
      K=K2
      NSTA(N-1)=K2
      IF(N.EQ.1) GO TO 880
      STOG(N-1)=ST
      N=N-1
      S(N,K)=ST
      GO TO 850
  880 X(1)= ST+R(N)-SEND
C WRITE THE OPTIMAL STORAGES AND RELEASES.
      WRITE(6,70)
   70 FORMAT(//5X,'MONTH',5X,'INFLOW',20X,'OPTIMAL REL.',20X,
     1'OPTIMAL STORAGE',10X,'STATE'//)
      DO 99 I=1,NS
      I1=601-I
      FL=R(I1)
      Z=XT(I1)
```

```
      Y=STOG(I1)
      M=NSTA(I1)
      WRITE(6,60)I,FL,Z,Y,M
   60 FORMAT(/5X,I5,F15.4,15X,F15.4,15X,F15.4,20X,I5)
   99 CONTINUE
      N1=601
      DO 50 I=1,50
      NSHT(I)=0
      DO 50 J=1,12
      N1=N1-1
      IF(XT(N1).LT.XTAP(N1)) NSHT(I)=NSHT(I)+1
      Q(I,J)=R(N1)
      Z1(I,J)=XT(N1)
      Y1(I,J)=STOG(N1)
   50 CONTINUE
      SUM7=0.0
      DO 62 J=1,12
      SUM6=0.0
      WRITE(6,63)J
   63 FORMAT(//30X,'MONTH IS',I5)
      WRITE(6,67)
   67 FORMAT(///10X,'YEAR',30X,'INFLOW',10X,'OPT.RELEASE',10X,
     1'OPT. STORAGE')
      DO 64 I=1,50
      T1=Q(I,J)
      T2=Z1(I,J)
      T3= Y1(I,J)
      WRITE(6,66)I,T1,T2,T3
   66 FORMAT(/10X,I5,20X,3F15.4)
      SUM6=SUM6+ Y1(I,J)
      SUM7=SUM7+Y1(I,J)
   64 CONTINUE
      AVST=(SUM6/50.0)
      WRITE(6,95)AVST
   95 FORMAT(//10X,'AV.STORAGE FOR THIS MONTH IS',F15.4//)
   62 CONTINUE
      TSTAV=(SUM7/600)
      WRITE(6,93)TSTAV
   93 FORMAT(//10X,'TOTAL AV.STORAGE OVER THE ENTIRE 50 YRS.',F15.4//)
      WRITE(6,77)(NSHT(I),I=1,50)
   77 FORMAT(//30X,'NO.SHORTAGES IN A YEAR'//(20X,10I5))
      GP=0.0
      DO 804 J=1,12
      GP=GP+1
      DO 803 I=1,48
      I2=50-I
      VAR(I,1)=Z1(I2,J)
      VAR(I,2)=Q(I2,J)
      VAR(I,3)=Y1(I2,J)
      I2=50-I
      M=J-1
      IF(M.LE.0)I2=I2-1
      IF(M.EQ.0) M=12
      VAR(I,4)=Q(I2,M)
      VAR(I,5)=Y1(I2,M)
      I2=50-I
      M=J-2
      IF(M.LE.0)I2=I2-1
      IF(M.LT.0) M=10+J
      IF(M.EQ.0) M=12
      VAR(I,6)=Q(I2,M)
      VAR(I,7)=Y1(I2,M)
```

```
      I2=50-I
      M=J-3
      IF(M.LE.0)I2=I2-1
      IF(M.LT.0) M=9+J
      IF(M.EQ.0) M=12
      VAR(I,8)= Q(I2,M)
      VAR(I,9)=Y1(I2,M)
      I2=50-I
      M=J-4
      IF(M.LE.0)I2=I2-1
      IF(M.LT.0) M=8+J
      IF(M.EQ.0) M=12
      VAR(I,10)=Q(I2,M)
      VAR(I,11)= Y1(I2,M)
      I2=50-I
      M=J-5
      IF(M.LE.0)I2=I2-1
      IF(M.LT.0)M=7+J
      IF(M.EQ.0) M=12
      VAR(I,12)=Q(I2,M)
      VAR(I,13)=Y1(I2,M)
  803 CONTINUE
  812 WRITE(6,813)J
  813 FORMAT(/30X,'MONTH IS',I5//8X,'REL.',1X,'INFLOW',1X,'STORAGE',
     115X,'INFLOWS AND STORAGES FROM LAG1 TO LAG5',2X//)
      DO 821 I=1,48
      WRITE(6,822) (VAR(I,K),K=1,13),GP
  822 FORMAT(5X,14F8.4)
      WRITE(3,875) (VAR(I,K),K=1,13),GP
  875 FORMAT(14F15.8)
  821 CONTINUE
  804 CONTINUE
  303 F=1
      G=2
      RETURN
      END
//GO.SYSIN DD *
53.0838   25.7308    2.168      75.0        0.50            52
   31    28    31    30    31    30    31    31    30    31    30    31
//GO.FT09F001 DD DSN=BHS500.BHASKAR.DATA,DISP=SHR
//GO.FT03F001 DD DSN=BHS500.DUNL6.DATA,DISP=(NEW,CATLG),UNIT=USERDA,
// DCB=(LRECL=210,BLKSIZE=2100,RECFM=FB),SPACE=(TRK,(15,2),RLSE)
/*
```

Simulation Program for Reservoir Operation

```
      DIMENSION QT(2000),JFL(200,12),STG(200,12),ND(12),NSHT(12),
     1BETA(12,12),OBJ(20,12),XTAR(12),PREL(200,12),REL(200,12),
     2PSHT(12),VSHT(12),UPP(12),DRELM(20,12),DDFM(20,12),DSTG(20,12),
     3ADRELM(12),ADDFM(12),ADSTG(12),AVNST(20,12),ANREL(12),ANSMX(12),
     4ANSMN(12),VNREL(12),VNSMX(12),VNSMN(12),NREL(20,12),NSMX(20,12),
     5NSMN(20,12),UPP1(12),UPP2(12),UPP3(12),AVOBJ(12),TREF(20),OLT(12),
     6SLT(12),EREL(12)
      REAL LOW(12),LOW1(12),LOW2(12),LOW3(12)
      DEFINE FILE 9(60,600,U,IFL)
C  READ IN THE INFLOW SEQUENCE.
C  DATA IS READ OFF A DISC FOR THIS SIMULATION.
C THREE GENERATED FLOW SEQUENCES OF 50 YEARS DURATION ARE SELECTED.
      NS=600
      NS1=NS+1
      NS2= NS+NS
      NS3=NS2+1
      NST=NS2+NS
      NYR=NST/12
      NREC=20
      IFL=1
C SET INDEX=0 FOR D.P.MODEL DERIVED FROM REGRESSION.
C  SET INDEX EQUAL=1 FOR STANDARD OPERATING POLICY.
C SET INDEX=2 FOR REVISED D.P.POLICY.
      INDEX=0
      WRITE(6,903)
  903 FORMAT(///30X,'SIMULATION OF HOOVER RESERVOIR MONTHLY RELEASE
     1POLICIES'////)
C  READ IN COEFFICIENTS FOR THE RELEASE POLICY.
C  WHERE THE VARIABLES ARE LESS THAN 7 SET THE MISSING VAR. COEFF=0.0.
      WRITE(6,9)
    9 FORMAT(///30X,'COEFFICIENTS OF VARIABLES IN THE RELEASE POLICY'//)
      DO 6 J=1,12
      READ(5,7) (BETA(J,K),K=1,7)
    7 FORMAT(7F10.0)
      WRITE(6,8) J,(BETA(J,K),K=1,7)
    8 FORMAT(/10X,I5,5X,7F15.6)
    6 CONTINUE
C  READ IN THE TARGET FOR EACH MONTH AND COMPUTE THE SHORTAGES IN
C  EACH MONTH
      READ(5,25) (ND(J),J=1,12),XPT
   25 FORMAT(12I5,F10.0)
C READ IN THE EXPECTED VALUES OF RELEASES FOR REVISED D.P.POLICY.
      READ(5,1450) (EREL(J),J=1,12)
 1450 FORMAT(6F10.0/6F10.0)
      WRITE(6,1451)(EREL(J),J=1,12)
 1451 FORMAT(///30X,'EXPECTED RELEASES FOR REVISED D.P.POLICY'////10X,
     112F10.6)
C READ ANY LOWER BOUNDS SET ON THE INDEPENDENT VARIABLES.
      READ(5,1002) (OLT(J),J=1,12)
      READ(5,1002) (SLT(J),J=1,12)
 1002 FORMAT(12F5.0)
      DO 900 LREC=1,NREC
      WRITE(6,901)LREC
  901 FORMAT(////40X,'DETAILS OF SIMULATION NUMBER',5X,I5////)
      WRITE(6,902)
  902 FORMAT(///40X,'EACH SIMULATION USES 150.0 YEARS OF INFLOW
     1DATA'////)
      IF(INDEX.NE.1) GO TO 1080
      WRITE(6,1070)
 1070 FORMAT(///40X,'SIMULATION OF THE STANDARD OPERATING POLICY'/////)
 1080 IF(INDEX.NE.2) GO TO 1090
      WRITE(6,1100)
```

```
 1100 FORMAT(////40X,'SIMULATION OF REVISED O.P.POLICY'/////)
 1090 G=1
      READ(9'IFL) (QT(I),I=1,NS)
      READ(9'IFL) (QT(I),I=NS1,NS2)
      READ(9'IFL) (QT(I),I=NS3,NST)
C  CONVERT THE INPUT INFLOW SEQ.
      K1=0
      DO 5 I=1,NYR
      DO 5 J=1,12
      K1=K1+1
      QFL(I,J)=QT(K1)
    5 CONTINUE
C  SET THE INITIAL CONDITIONS.
      NYR1=NYR+1
      DO 4 I=1,NYR1
      DO 4 J=1,12
      REL(I,J)=0.0
      STG(I,J)=0.0
    4 CONTINUE
C  CONVERT THE TARGET FROM MGD. TO TH.AC.FT./MONTH.
      DO 30 J=1,12
      XTAR(J)=((XPT*ND(J)/0.326)/1000.0)
   30 CONTINUE
      SMIN=2.1860
      SMAX=60.3420
      STG(2,1)=SMAX
C ---------INPUT THE RELEASE MODEL------------------
      NREL(LREC,J)=0
      NSMX(LREC,J)=0
      NSMN(LREC,J)=0
      DO 10 I=2,NYR
      DO 16 J=1,12
      J1=J-1
      I1=I
      IF(J1.EQ.0) I1=I-1
      IF(J1.EQ.0) J1=12
      K1=J-2
      I3=I
      IF(K1.LE.0)I3=I-1
      IF(K1.EQ.0)K1=12
      IF(K1.LT.0)K1=11
      QSP=QFL(I,J)+STG(I,J)
      QSPS=(QFL(I,J)+STG(I,J))**2
      QSPQ=(QFL(I,J)+STG(I,J))**3
      CRP=QFL(I,J)*STG(I,J)
      QSS=QFL(I,J)**2
      STS=STG(I,J)**2
      IF(INDEX.EQ.1) GO TO 1060
      IF(INDEX.EQ.2) GO TO 1350
      REL(I,J)=BETA(J,1)+BETA(J,2)*QSP+BETA(J,3)*QSPS+BETA(J,4)*QSPQ
      GO TO 1050
 1060 CALL STANOP(I,J,LREC,QFL,STG,REL,SMAX,SMIN,XTAR,NSMX,BETA)
      GO TO 1050
 1350 CALL REVOP(I,J,LREC,QFL,STG,REL,SMAX,SMIN,EREL,NSMX)
 1050 PREL(I,J)=REL(I,J)
      IF(REL(I,J).LT.0.0) NREL(LREC,J)=NREL(LREC,J)+1
      IF(I.EQ.2) NREL(LREC,J)=0
      IF(REL(I,J).LT.0.0) REL(I,J)=0.0
C  COMPUTE THE END STORAGE AND MAKE ADJUSTMENTS FOR NEGATIVE PREDICTED
C  RELEASES OR STORAGE FALLING OUT OF THE PRESCRIBED BOUNDRIES.
      J2=J+1
```

IJ5I

```
      IF(J2.EQ.13) I2=I+1
      IF(J2.EQ.13) J2=1
      STG(I2,J2)=STG(I,J)+QFL(I,J) -REL(I,J)
      IF(STG(I2,J2).LT.SMIN) GO TO 200
      IF(STG(I2,J2).GT.SMAX) GO TO 300
      GO TO 15
  200 NSMN(LREC,J)=NSMN(LREC,J)+1
      IF(I.EQ.2) NSMN(LREC,J)=0
      STG(I2,J2)=SMIN
      REL(I,J)=STG(I,J)+QFL(I,J)-STG(I2,J2)
      GO TO 15
  300 NSMX(LREC,J)=NSMX(LREC,J)+1
      IF(I.EQ.2) NSMX(LREC,J)=0
      STG(I2,J2)=SMAX
      REL(I,J)=STG(I,J) +QFL(I,J)-STG(I2,J2)
   15 IT=I-1
      P1=PREL(I,J)
      R1=REL(I,J)
      Q1=QFL(I,J)
      Q2=QFL(I1,J1)
      Q3=QFL(I3,K1)
      S1=STG(I,J)
   16 CONTINUE
   10 CONTINUE
C  WRITE ALL DATA ON DISC FOR STATISTICAL ANALYSIS.
      GP=0.0
      IF(LREC.GT.1) GO TO 1205
      DO 1120 J=1,12
      GP=GP+1.0
      DO 1125 I=3,NYR
      J1=J-1
      I1=I
      IF(J1.EQ.0) I1=I-1
      IF(J1.EQ.0)J1=12
      K1=J-2
      I3=I
      IF(K1.LE.0)I3=I-1
      IF(K1.EQ.0)K1=12
      IF(K1.LT.0)K1=11
      P1=PREL(I,J)
      R1=REL(I,J)
      S1=STG(I,J)
      Q1=QFL(I,J)
      Q2=QFL(I1,J1)
      Q3=QFL(I3,K1)
 1125 CONTINUE
 1120 CONTINUE
 1205 F=1
      DO 27 J=1,12
      SUMR=0.0
      SUMQ=0.0
      SUMS=0.0
      DO 20 I=3,NYR
      SUMR=SUMR+REL(I,J)
      SUMQ=SUMQ+QFL(I,J)
      SUMS=SUMS+STG(I,J)
      J1=J-1
      I1=I
      IF(J1.EQ.0) I1=I-1
      IF(J1.EQ.0) J1=12
      P1=PREL(I,J)
      R1=REL(I,J)
```

```
      Q1=QFL(I,J)
      Q2=QFL(I1,J1)
      S1=STG(I,J)
      IT=I-2
   20 CONTINUE
      NYR2=NYR-2
      DRELM(LREC,J)=SUMR/NYR2
      DQFM(LREC,J)=SUMQ/NYR2
      DSTG(LREC,J)=SUMS/NYR2
   27 CONTINUE
      WRITE(6,604)
  604 FORMAT(//10X,'MONTH',10X,'NO.OF RELEASES<0.0',10X,'NO.OF MAX.
     1STG.EXCEEDED',10X,'NO. OF MIN.STG.EXCEEDED'///)
      DO 603 J=1,12
      N1=NREL(LREC,J)
      N2=NSMX(LREC,J)
      N3=NSMN(LREC,J)
      WRITE(6,667) J,N1,N2,N3
  667 FORMAT(10X,I5,20X,I5,20X,I5,20X,I5)
  603 CONTINUE
C  COMPUTE THE STATISTICS ON THE STOCHASTIC VARIABLES----INFLOW,
C  STORAGE AND RELEASE.
      WRITE(6,805)
  805 FORMAT(//30X,'EXPECTED VALUES OF STOCHASTIC VARIABLES'///10X,
     1'MONTH',20X,'MEAN REL.',15X,'MEAN INFLOW',15X,'MEAN STORG.'///)
      DO 801 J=1,12
      E1=DRELM(LREC,J)
      E2=DQFM(LREC,J)
      E3=DSTG(LREC,J)
      WRITE(6,802)J,E1,E2,E3
  802 FORMAT(10X,I5,20X,F15.6,10X,F15.6,10X,F15.6)
  801 CONTINUE
      WRITE(6,31)
   31 FORMAT(//30X,'MONTHLY TARGET RELEASES IN TH.AC.FT.'///)
      WRITE(6,32) (XTAR(J),J=1,12)
   32 FORMAT(10X,6F15.6//10X,6F15.6)
C  COMPUTE THE OBJECTIVE FUNCTION VALUE.
C  THE CRITERION USED FOR COMPARISON OF POLICIES SUM OF SQUARED
C  DEVIATIONS OF RELEASES FROM THE TARGET.
      SUM=0.0
      DO 40 J=1,12
      SUM1=0.0
      NSHT(J)=0.0
      DO 45 I=3,NYR
      SUM= SUM+ (REL(I,J)-XTAR(J))**2
      SUM1= SUM1 + (REL(I,J)-XTAR(J))**2
      IF(REL(I,J).LT.XTAR(J)) NSHT(J) =NSHT(J) +1
   45 CONTINUE
      OBJ(LREC,J)=SUM1
   40 CONTINUE
      TRET(LREC)=SUM
      WRITE(6,50)
   50 FORMAT(//10X,'MONTH',20X,'TOTAL RET./MO.',20X,'NO.OF SHORTAGES
  501PER MONTH'///)
      DO 60 J=1,12
      N1=NSHT(J)
      OP=OBJ(LREC,J)
      WRITE(6,61)J,OP,N1
   61 FORMAT(10X,I5,20X,F15.4,30X,I5)
   60 CONTINUE
      WRITE(6,666) TRET(LREC)
  666 FORMAT(//30X,'TOTAL RETURN IN THE ENTIRE OPERATION',10X,F15.6///)
```

```
C  COMPUTE STATISTICS ON THE MONTHLY SHORTAGES.
      WRITE(6,409)
  409 FORMAT(///30X,'AVERAGE NO.OF SHORTAGES /YEAR IN A MONTH'//10X,
     1'SEQ.NO.',10X,'MONTH',20X,'AV.NO.OF SHORTG./YR.'///)
      NYR1=NYR-2
      DO 410 J=1,12
      AVNST(LREC,J)=(1.0*NSHT(J))/NYR1
      A1=AVNST(LREC,J)
      WRITE(6,411)LREC,J,A1
  411 FORMAT(10X,I5,10X,I5,20X,F15.6)
  410 CONTINUE
  900 CONTINUE
      WRITE(6,999)
  999 FORMAT(/////40X,'OVERALL STATISTICS FROM ALL GEN. SEQUENCES'////)
      DO 430 J=1,12
      SUM1=0.0
      SUM2=0.0
      DO 440 I=1,NREC
      SUM1=SUM1+ AVNST(I,J)
      SUM2=SUM2+ (AVNST(I,J)**2)
  440 CONTINUE
      NREC1=NREC-1
      PSHT(J)=SUM1/NREC
      VSHT(J)=(SUM2/NREC1)-((NREC/NREC1)*(PSHT(J))**2)
  430 CONTINUE
C  COMPUTE THE CONFIDENCE INTERVALS FOR THE AV. NO. OF SHORTAGES/MO.
C  USE THE CRITICAL VALUE FROM THE NORMAL DISTRIBUTION.
      WRITE(6,98)
   98 FORMAT(//30X,'CONFIDENCE INTERVAL ON THE AV.NO.OF SHORTAGES
     1IN A MONTH'//10X,'MONTH',10X,'UPPER 95% C.I.',10X,'LOWER 95%
     2C.I.',15X,'MEAN',15X,'VARIANCE'///)
C  THE CONFIDENCE LIMITS ARE OBTAINED USING 'T' DISTRIBUTION
C  WITH THE CRITICAL VALUE AT 95% LEVEL AND 19 D.F.
      Z1=2.093
      DO 90 J=1,12
      UPP(J)=PSHT(J) + Z1*SQRT(VSHT(J)/NREC)
      LOW(J)=PSHT(J) - Z1*SQRT(VSHT(J)/NREC)
      V1=PSHT(J)
      V2=VSHT(J)
      UT=UPP(J)
      PT=LOW(J)
      WRITE(6,91)J,UT,PT,V1,V2
   91 FORMAT(10X,I5,10X,F15.6,10X,F15.6,10X,F15.6,10X,F15.6)
   90 CONTINUE
      WRITE(6,873)
  873 FORMAT(///30X,'AVERAGES OF EXPECTED VALUES OF THE STOCHASTIC
     1VARIABLES'///10X,'MONTH',20X,'AV.RELEASE',10X,'AV.INFLOW',10X,'AV.
     2STORAGE'//)
      NYR1=NYR-2
      DO 871 J=1,12
      SUMR=0.0
      SUMQ=0.0
      SUMS=0.0
      SUMA=0.0
      SUMB=0.0
      SUMC=0.0
      SUMD=0.0
      SUME=0.0
      SUMF=0.0
      DO 872 I=1,NREC
      SUMD=SUMD+ (1.0*NREL(I,J)/NYR1)**2
      SUME=SUME+ (1.0*NSMX(I,J)/NYR1)**2
```

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      SUMF=SUMF+ (1.0*NSMN(I,J)/NYR1)**2
      SUMA=SUMA+(1.0*NREL(I,J)/NYR1)
      SUMB=SUMB+(1.0*NSMX(I,J)/NYR1)
      SUMC=SUMC+(1.0*NSMN(I,J)/NYR1)
      SUMQ=SUMQ+DQFM(I,J)
      SUMS=SUMS+DSTG(I,J)
      SUMR=SUMR+DRELM(I,J)
  872 CONTINUE
      ANREL(J)=SUMA/NREC
      ANSMX(J)=SUMB/NREC
      ANSMN(J)= SUMC/NREC
      VNREL(J)=(SUMD/NREC1)-((NREC/NREC1)*(ANREL(J))**2)
      VNSMX(J)= (SUME/NREC1) -((NREC/NREC1)*(ANSMX(J))**2)
      VNSMN(J)= (SUMF/NREC1)- ((NREC/NREC1)*(ANSMN(J))**2)
      ADRELM(J)=SUMR/NREC
      ADQFM(J)=SUMQ/NREC
      ADSTG(J)=SUMS/NREC
      A1=ADRELM(J)
      A2=ADQFM(J)
      A3=ADSTG(J)
      WRITE(6,874)J,A1,A2,A3
  874 FORMAT(10X,I5,20X,F15.6,10X,F15.6,10X,F15.6)
  871 CONTINUE
C  COMPUTE THE CONFIDENCE INTERVALS ON THE NUMBER OF VIOLATIONS
C  OF NEGATIVE RELEASE ,MAX.STORAGE AND MIN. STORAGE EXCEEDENCES.
      WRITE(6,710)
  710 FORMAT(///30X,'CONFIDENCE INTERVAL ON VIOLATIONS OF POST.REL.,MAX.
     1AND MIN.STORAGES'///)
      DO 709 J=1,12
      UPP1(J)=ANREL(J)+Z1*SQRT(VNREL(J)/NREC)
      LOW1(J)=ANREL(J)-Z1*SQRT(VNREL(J)/NREC)
      UPP2(J)= ANSMX(J)+ Z1*SQRT(VNSMX(J)/NREC)
      LOW2(J)= ANSMX(J)- Z1*SQRT(VNSMX(J)/NREC)
      UPP3(J)=ANSMN(J)+ Z1*SQRT(VNSMN(J)/NREC)
      LOW3(J)=ANSMN(J)- Z1*SQRT(VNSMN(J)/NREC)
  709 CONTINUE
      WRITE(6,981)
  981 FORMAT(///30X,' STATISTICS ON AVERAGE RELEASE VIOLATIONS'//)
      WRITE(6,712)
  712 FORMAT(//10X,'MONTH',10X,'UPPER 95% C.I.',10X,'LOWER 95%C.I.',
     110X,'MEAN',15X,'VARIANCE'///)
      DO 982 J=1,12
      WRITE(6,711) J,UPP1(J),LOW1(J),ANREL(J),VNREL(J)
  711 FORMAT(10X,I5,4(10X,F12.4))
  982 CONTINUE
      WRITE(6,983)
  983 FORMAT(///30X,'STATISTICS ON MAX.STORAGE VIOLATIONS'///)
      WRITE(6,712)
      DO 984 J=1,12
      WRITE(6,711)J,UPP2(J),LOW2(J),ANSMX(J),VNSMX(J)
  984 CONTINUE
      WRITE(6,985)
  985 FORMAT(///30X,'STATISTICS ON MIN.STORAGE VIOLATIONS'///)
      WRITE(6,712)
      DO 986 J=1,12
      WRITE(6,711) J,UPP3(J),LOW3(J),ANSMN(J),VNSMN(J)
  986 CONTINUE
      WRITE(6,935)
  935 FORMAT(///30X,'STATISTICS ON THE AVERAGE MONTHLY RETURNS'///10X,
     1'MONTH',20X,'RETURN'///)
      DO 930 J=1,12
      SUMU=0.0
```

```
      DO 932 I=1,NREC
      SUMO=SUMO+OBJ(I,J)
  932 CONTINUE
      AVOBJ(J)= SUMO/NREC
      AO=AVOBJ(J)
      WRITE(6,933)J,AO
  933 FORMAT(10X,I5,20X,F15.6)
  930 CONTINUE
      SUMY=0.0
      DO 941 I=1,NREC
      SUMY=SUMY+TRET(I)
  941 CONTINUE
      ATRET=SUMY/NREC
      WRITE(6,808)ATRET
  808 FORMAT(///30X,'TOTAL AVERAGE RETURN PER YEAR=',5X,F15.6///)
      STOP
      END
      SUBROUTINE STANOP(I,J,LREC,QFL,STG,REL,SMAX,SMIN,XTAR,NSMX,BETA)
      DIMENSION QFL(200,12),STG(200,12),REL(200,12),XTAR(12),
     1NSMX(20,12),BETA(12,12)
C  THIS PROGRAM COMPUTES RELEASES USING THE STANDARD OPERATING POLICY.
      SUM=STG(I,J)+QFL(I,J)
      SUM1=SUM
      SUM2=SUM**2
      SUM3=SUM**3
      RSM=SMAX+XTAR(J)
      IF(SUM.LE.RSM) GO TO 50
C COMPUTE THE SPILL.
      REL(I,J)=SUM-SMAX
      NSMX(LREC,J)=NSMX(LREC,J)+1
      RETURN
   50 IF(SUM.LT.XTAR(J)) GO TO 60
C RELEASE THE TARGET AMOUNT.
C IF REVISED D.P. FROM REGRESSION IS USED THEN SET RELEASE AS
C  PREDICTED BY THE POLICY.
      J1=J-1
      I1=I-1
      IF(J1.EQ.0)I1=I-1
      IF(J1.EQ.0)J1=12
      K1=J-2
      I3=I
      IF(K1.LE.0) I3=I-1
      IF(K1.EQ.0)K1=12
      IF(K1.LT.0)K1=11
      REL(I,J)=BETA(J,1)+BETA(J,2)*SUM1+BETA(J,3)*SUM2+BETA(J,4)*SUM3
      RETURN
C EMPTY THE RESERVOIR TILL THE DEAD STORAGE LEVEL.
   60 REL(I,J)=SUM-SMIN
      RETURN
      END
      SUBROUTINE REVDP(I,J,LREC,QFL,STG,REL,SMAX,SMIN,EREL,NSMX)
      DIMENSION QFL(200,12),STG(200,12),REL(200,12),EREL(12),
     1NSMX(20,12)
C THIS PROGRAM COMPUTES RELEASES USING REVISED D.P. POLICY.
      SUM=STG(I,J)+QFL(I,J)
      RSM=SMAX+EREL(J)
      IF(SUM.LE.RSM) GO TO 50
C COMPUTE THE SPILL.
      REL(I,J)=SUM-SMAX
      NSMX(LREC,J)=NSMX(LREC,J)+1
      RETURN
   50 IF(SUM.LT.EREL(J)) GO TO 60
```

```
C RELEASE THE TARGET AMOUNT.
      REL(I,J)=EPEL(J)
      RETURN
C EMPTY THE RESERVOIR TILL THE DEAD STORAGE LEVEL.
   60 REL(I,J)=SUM-SMIN
      RETURN
      END
9.47463745          0.00306614
9.44500134                    0.00002913
11.3431967          -.002733450.00005272
13.5809511          -.0060563 0.00008582
17.1959655          -.010960470.00014831
11.0388017          -.004108010.00006402
-21.0684151.83336718-.036506520.00024407
-15.4020121.67576527-.036327590.00025926
9.145285890.04548679
10.067
10.121
7.97954727          0.00581901
   31   28   31   30   31   30   31   31   30   31   30   31 100.0
11.535521611.494504111.578523 10.152295 10.184482 9.544552
9.795946  9.930805  9.9432806 10.067254410.175509711.6153648
365.0440.0473.0456.0315.0325.0198.0273.0 0.0  0.0  0.0  244.0
     30.0 37.0 45.0
```

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